Bengt Langesten

Byggkonstruktion 1

Lösningar till övningsuppgifter

Byggnadsstatik

Liber

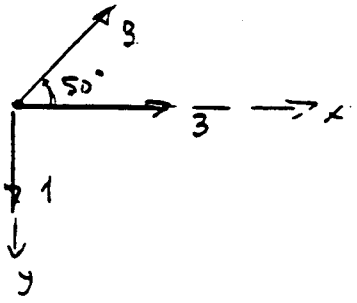
ISBN 91-47-00654-4

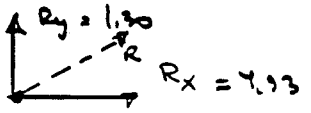
Andra upplagan 3

Tryck:
Novum Grafiska
Stockholm 2001

Liber AB, 113 98 Stockholm
tfn 08-690 92 00
www.liber.se
kundtjänst tfn 08-690 93 30, fax 08-690 93 01,
e-post: kundtjanst.liberab@liber.se

Ex 1





$R_x = 3 + 3\cos 50 = 4{,}93$ kN

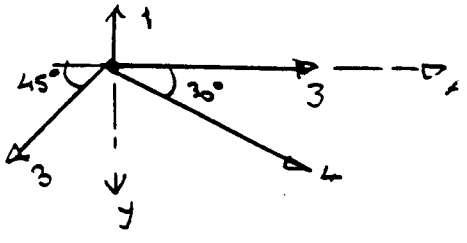
$R_y = 1 - 3\sin 50 = -1{,}30$ kN

$R = \sqrt{4{,}93^2 + 1{,}30^2} = \underline{5{,}10 \text{ kN}}$

$\tan\alpha = \frac{1{,}30}{4{,}93}$

$\underline{\alpha = 14{,}8^\circ}$ mot uppåt höger

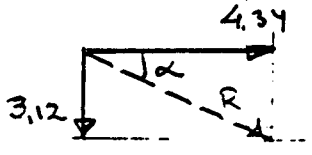
Ex 2



$R_x = 3 + 4\cdot\cos 30 - 3\cos 45 = 4{,}34$ kN

$R_y = -1 + 4\cdot\sin 30 + 3\cdot\sin 45 = 3{,}12$

$R = \sqrt{4{,}34^2 + 3{,}12^2} = \underline{5{,}35 \text{ kN}}$



$\tan\alpha = \frac{3{,}12}{4{,}34}$

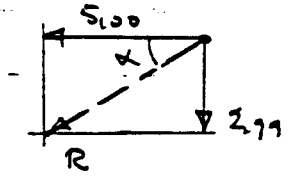
$\underline{\alpha = 35{,}7^\circ}$

$\underline{\text{mot vänster höger}}$

Ex 3

$R_x = 3\cdot\cos 20 + 8\cdot\cos 70 - 7\cos 40 - 6\cos 30 = -5{,}00$ kN

$R_y = -3\sin 20 + 8\sin 70 + 7\sin 40 - 6\sin 30 - 5 = +2{,}99$ kN



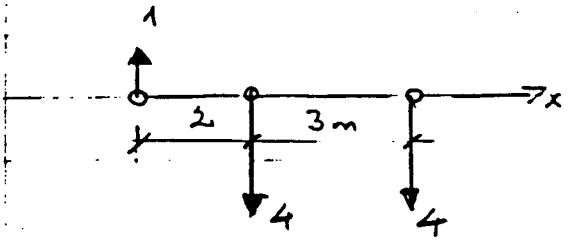
$R = \sqrt{5{,}00^2 + 2{,}99^2} = \underline{5{,}83 \text{ kN}}$

$\tan\alpha = \frac{2{,}99}{5{,}00}$

$\underline{\alpha = 30{,}9^\circ}$

$\underline{\text{mot vänster vänster}}$

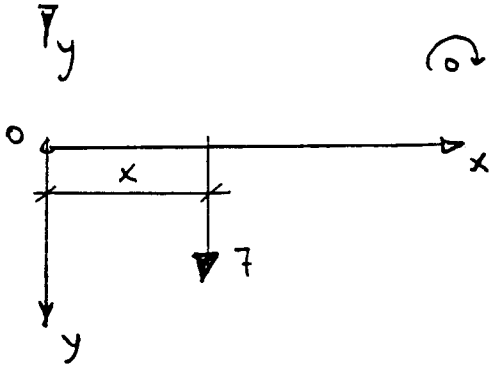
Ex 4



$R_x = 0$

$R_y = 7$ kN

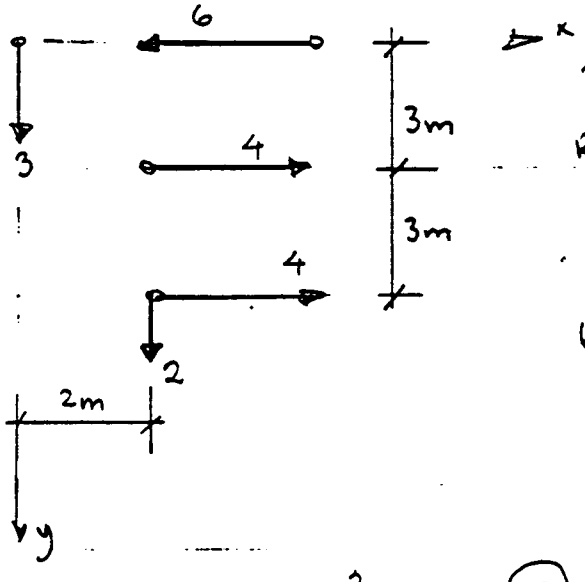
ger $R = 7$ kN nedåt



$\circlearrowleft_0$ $7 \cdot x = 4 \cdot 2 + 4 \cdot 5$

varav $x = 4$ m till höger om 1 kN-kraften

Ex 5



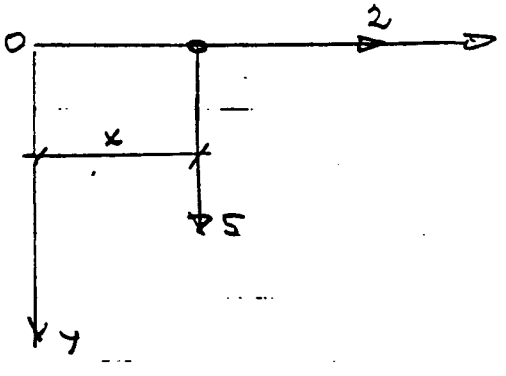
$R_x = -6 + 4 + 4 = 2$ kN

$R_y = 3 + 2 = 5$ kN

$R = \sqrt{2^2 + 5^2} = 5{,}39$ kN

$\tan\alpha = \frac{5}{2}$

$\alpha = 68{,}2°$ mot nedåt höger

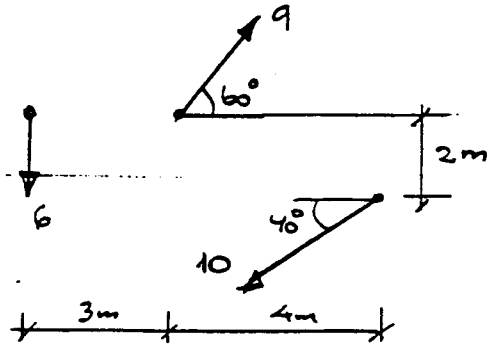


$\circlearrowleft_0$ $5 \cdot x = 2 \cdot 2 - 4 \cdot 3 - 4 \cdot 6$

$x = -6{,}4$ m

d.v.s. 6,4 m till vänster om 3-kN-kraftens angreppspunkt

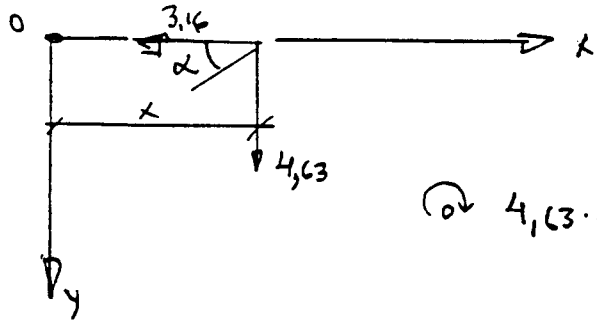
Ex 6



$R_x = 9 \cdot \cos 60 - 10 \cos 40 = -3{,}16$ kN

$R_y = 6 - 9 \sin 60 + 10 \cdot \sin 40 = 4{,}63$ kN

$R = \sqrt{3{,}16^2 + 4{,}63^2} = \underline{5{,}61 \text{ kN}}$



$\tan\alpha = \frac{4{,}63}{3{,}16}$ $\quad \underline{\alpha = 55{,}7}$ vrett

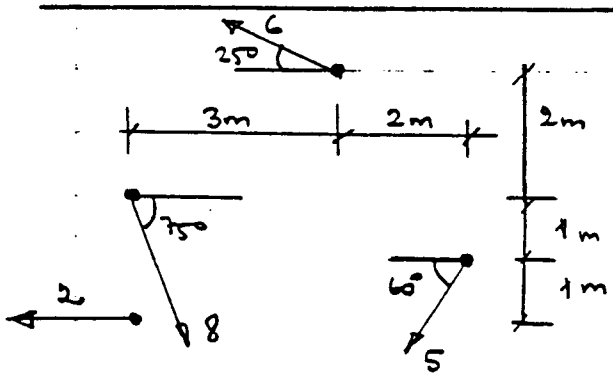
vänster

$\circlearrowright_0 \quad 4{,}63 \cdot x = -9 \cdot \sin 60 \cdot 3 + 10 \cos 40 \cdot 2 + 10 \sin 40 \cdot 7$

$x = \frac{36{,}93}{4{,}63} = \underline{7{,}98 \text{ m}}$ till höger

om 6-kN-kraftens angreppspunkt

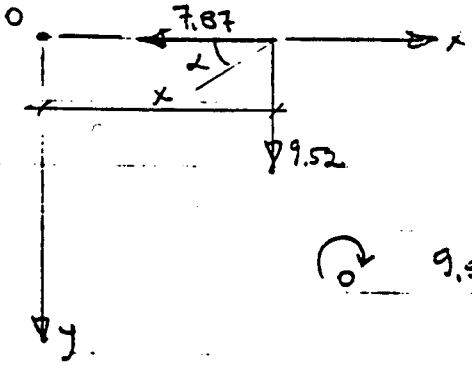
Ex 7



$R_x = -2 - 6 \cos 25 + 8 \cdot \cos 75 - 5 \cdot \cos 60 = -7{,}87$

$R_y = -6 \sin 25 + 8 \cdot \sin 75 + 5 \cdot \sin 60 = 9{,}52$

$R = \sqrt{7{,}87^2 + 9{,}52^2} = \underline{12{,}35 \text{ kN}}$



$\tan\alpha = \frac{9{,}52}{7{,}87}$ $\quad \underline{\alpha = 50{,}4°}$ vrett

vänster

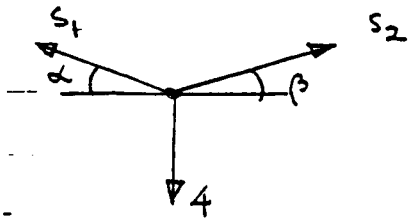
$\circlearrowright_0 \quad 9{,}52 \cdot x = 9 \cdot 2 - 6 \cos 25 \cdot 2 - 6 \sin 25 \cdot 3$

$+ 5 \cos 60 \cdot 1 + 5 \cdot \sin 60 \cdot 5$

$x = \frac{9{,}67}{9{,}52} = \underline{1{,}02 \text{ m}}$ till höger om

8 kN-kraftens angreppspunkt

Ex 8



$\tan\alpha = \frac{2}{7}$ $\quad \alpha = 15{,}9°$

$\tan\beta = \frac{2}{5}$ $\quad \beta = 21{,}8°$

$\rightarrow \quad -S_1 \cdot \cos 15{,}9 + S_2 \cdot \cos 21{,}8 = 0$

$\downarrow \quad 4 - S_1 \cdot \sin 15{,}9 - S_2 \cdot \sin 21{,}8 = 0$

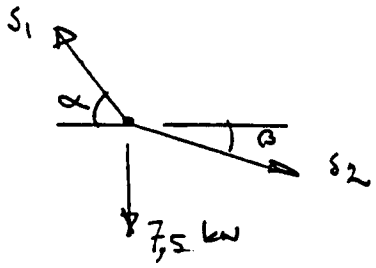
Första ekv ger $\quad S_2 = \frac{S_1 \cdot \cos 15{,}9}{\cos 21{,}8} = 1{,}036 \cdot S_1$

$4 - S_1 \cdot \sin 15{,}9 - 1{,}036\, S_1 \cdot \sin 21{,}8 = 0$

$S_1 = \frac{4}{0{,}659} = 6{,}07 \text{ kN} = \underline{6{,}1 \text{ kN}}$

$S_2 = 1{,}036 \cdot 6{,}07 = 6{,}29 \text{ kN} = \underline{6{,}3 \text{ kN}}$

Ex 9



$\tan\alpha = \frac{30}{20}$ $\quad \alpha = 56{,}3°$

$\tan\beta = \frac{20}{80}$ $\quad \beta = 14{,}0°$

$\rightarrow \quad -S_1 \cdot \cos 56{,}3 + S_2 \cdot \cos 14{,}0 = 0$

$\downarrow \quad 7{,}5 - S_1 \cdot \sin 56{,}3 + S_2 \cdot \sin 14{,}0 = 0$

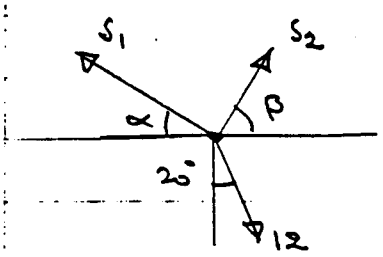
Första ekv ger $\quad S_2 = \frac{S_1 \cdot \cos 56{,}3}{\cos 14} = 0{,}572 \cdot S_1$

$7{,}5 - S_1 \cdot \sin 56{,}3 + 0{,}572 \cdot S_1 \cdot \sin 14 = 0$

$S_1 = \frac{7{,}5}{0{,}6935} = \underline{10{,}8 \text{ kN}}$

$S_2 = 0{,}572 \cdot 10{,}8 = \underline{6{,}2 \text{ kN}}$

Ex 10



$$\tan\alpha = \frac{2}{3} \qquad \alpha = 33{,}7^\circ$$

$$\tan\beta = \frac{2}{1} \qquad \beta = 63{,}4^\circ$$

$$\rightarrow \quad -S_1 \cdot \cos 33{,}7 + S_2 \cdot \cos 63{,}4 + 12 \cdot \sin 20^\circ = 0$$

$$\uparrow \quad S_1 \cdot \sin 33{,}7 + S_2 \cdot \sin 63{,}4 - 12 \cdot \cos 20 = 0$$

$$S_2 = \frac{S_1 \cdot \cos 33{,}7 - 4{,}10}{\cos 63{,}4} = 1{,}86\, S_1 - 9{,}17$$

$$S_1 \cdot \sin 33{,}7 + (1{,}86\, S_1 - 9{,}17) \cdot \sin 63{,}4 = 11{,}28$$

$$S_1 (0{,}555 + 1{,}663) = 11{,}28 + 8{,}20$$

$$S_1 = \frac{19{,}48}{2{,}218} = 8{,}78 = \underline{8{,}8 \text{ kN}}$$

$$S_2 = 1{,}86 \cdot 8{,}78 - 9{,}17 = 7{,}17 = \underline{7{,}2 \text{ kN}}$$

Ex 11

$$\rightarrow \quad -S_1 \cdot \cos 33{,}7 + S_2 \cdot \cos 63{,}4 - 12 \cdot \sin 20 = 0$$

$$\uparrow \quad S_1 \cdot \sin 33{,}7 + S_2 \cdot \sin 63{,}4 - 12 \cos 20 = 0$$

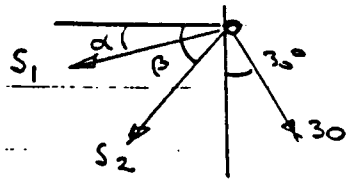
$$S_2 = \frac{S_1 \cos 33{,}7 + 4{,}10}{\cos 63{,}4} = 1{,}86\, S_1 + 9{,}17$$

$$S_1 \sin 33{,}7 + (1{,}86\, S_1 + 9{,}17) \cdot \sin 63{,}4 = 11{,}28$$

$$S_1 = \frac{3{,}08}{2{,}218} = 1{,}39 \text{ kN} = \underline{1{,}4 \text{ kN}}$$

$$S_2 = 1{,}86 \cdot 1{,}39 + 9{,}17 = \underline{11{,}7 \text{ kN}}$$

Ex 12



$$\tan\alpha = \frac{6}{8} \qquad \alpha = 36{,}9°$$

$$\tan\beta = \frac{6}{3} \qquad \alpha = 63{,}4°$$

$$\longrightarrow \quad -S_1 \cdot \cos 36{,}9 - S_2 \cdot \cos 63{,}4 + 30 \cdot \sin 30 = 0$$

$$\downarrow \quad S_1 \sin 36{,}9 + S_2 \cdot \sin 63{,}4 + 30 \cdot \cos 30 = 0$$

$$S_2 = \frac{15 - S_1 \cdot \cos 36{,}9}{\cos 63{,}4} = 33{,}5 - 1{,}786 S_1$$

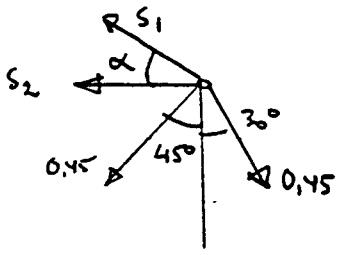
$$S_1 \cdot \sin 36{,}9 + \left(33{,}5 - 1{,}786 S_1\right) \cdot \sin 63{,}4 + 26{,}0 = 0$$

$$-0{,}997 S_1 = -26{,}0 - 29{,}95$$

$$\underline{S_1 = +56 \text{ kN}}$$

$$S_2 = 33{,}5 - 1{,}786 \cdot 56 = \underline{-67 \text{ kN}}$$

Ex 13



$$\tan\alpha = \frac{1{,}6}{2{,}5} \qquad \alpha = 32{,}6$$

$$\longrightarrow \quad -S_1 \cdot \cos 32{,}6° - S_2 - 0{,}45 \cdot \sin 45° + 0{,}45 \cdot \sin 30° = 0$$

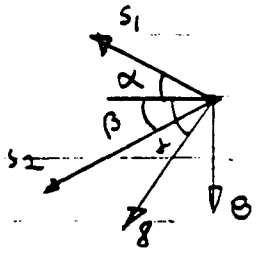
$$\downarrow \quad -S_1 \sin 32{,}6° + 0{,}45 \cdot \cos 45 + 0{,}45 \cdot \cos 30 = 0$$

Den under elv ser

$$S_1 = \frac{0{,}708}{\sin 32{,}6} = 1{,}314 \text{ kN} = \underline{1{,}31 \text{ kN}}$$

$$S_2 = -1{,}314 \cos 32{,}6 - 0{,}093 = \underline{-1{,}20 \text{ kN}}$$

Ex 14



$$\tan\alpha = \frac{1.4}{4} \qquad \alpha = 19{,}3^\circ$$

$$\tan\beta = \frac{3}{4} \qquad \beta = 36{,}9^\circ$$

$$\tan\gamma = \frac{5}{3} \qquad \gamma = 59{,}0^\circ$$

$\leftarrow$ $S_1 \cdot \cos 19{,}3 + S_2 \cdot \cos 36{,}9 + 8 \cdot \cos 59 = 0$

$\downarrow$ $-S_1 \sin 19{,}3 - S_2 \cdot \sin 36{,}9 + 8 \cdot \sin 59 + \theta = 0$

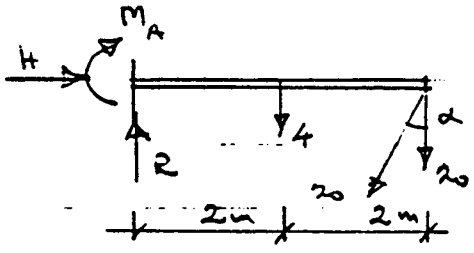
$$S_2 = \frac{-4{,}12 - S_1 \cdot \cos 19{,}3}{\cos 36{,}9} = -5{,}15 - 1{,}18\, S_1$$

$$-S_1 \cdot \sin 19{,}3 - (5{,}15 + 1{,}18\, S_1) \sin 36{,}9 + 14{,}86 = 0$$

$$S_1 = \frac{11{,}77}{1{,}039} = \underline{11{,}3 \text{ kN}}$$

$$S_2 = -5{,}15 - 1{,}18 \cdot 11{,}3 = \underline{-18{,}5 \text{ kN}}$$

Ex 15



$$\tan\alpha = \frac{2}{5} \qquad \alpha = 21{,}8^\circ$$

$\rightarrow$ $H - 20 \cdot \sin 21{,}8 = 0$

$\uparrow$ $R - 20 - 4 - 20 \cos 21{,}8 = 0$

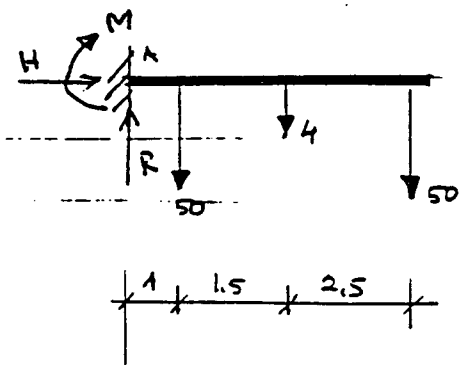
$\circlearrowleft A$ $M_A + 4 \cdot 2 + (20 + 20 \cos 21{,}8) = 0$

som ger

$\underline{H = 7{,}4 \text{ kN}}$

$\underline{R = 42{,}7 \text{ kN}}$

$\underline{M_A = -162 \text{ kNm}}$



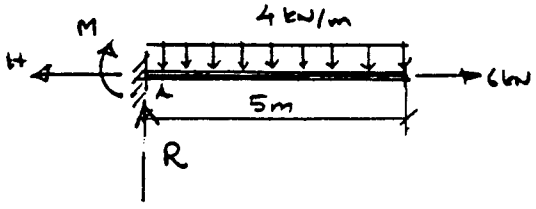
Ex 16

$\rightarrow$ $H = 0$

$\uparrow$ $R - 104 = 0$ $\quad$ $R = 104$ kN

$\circlearrowright A$ $M + 50 \cdot 1 + 4 \cdot 2{,}5 + 50 \cdot 5 = 0$

$M = -310$ kNm



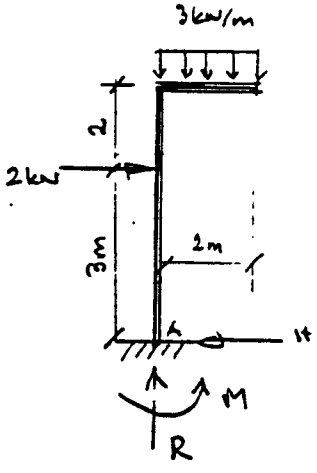
Ex 17

$\leftarrow$ $H - 6 = 0$ $\quad$ $H = 6$ kN

$\uparrow$ $R - 4 \cdot 5 = 0$ $\quad$ $R = 20$ kN

$\circlearrowright A$ $M + 4 \cdot 5 \cdot 2{,}5 = 0$ $\quad$ $M = -50$ kNm

---

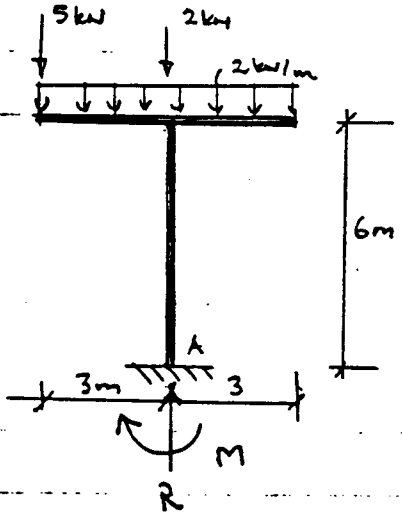


Ex 18

$\rightarrow$ $2 - H = 0$ $\quad$ $H = 2$ kN

$\uparrow$ $R - 3 \cdot 2 = 0$ $\quad$ $R = 6$ kN

$\circlearrowleft A$ $M - 2 \cdot 3 - 3 \cdot 2 \cdot 1 = 0$ $\quad$ $M = 12$ kNm (d.v.s.)



Ex 19

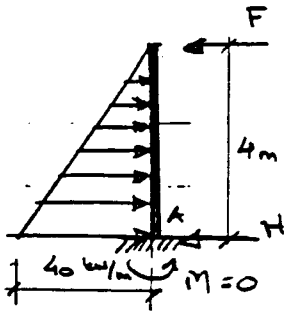
$\uparrow R - 5 - 2 - 2 \cdot 6 = 0$

$\circlearrowright A$ $M - 5 \cdot 3 = 0$

$R = 19$ kN

$M = 15$ kNm (d.h.s.)

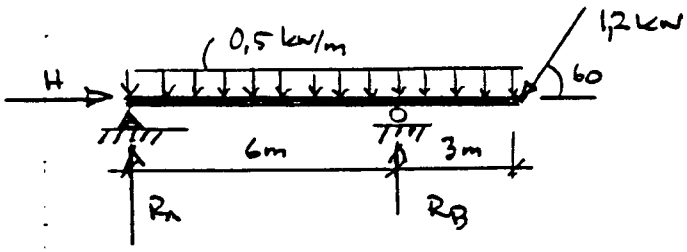
Ex 20



$\circlearrowleft A \quad F \cdot 4 - \frac{40 \cdot 4}{2} \cdot \frac{4}{3} = 0$

$F = 26{,}7 \text{ kN}$

Ex 21



$\rightarrow \quad H - 1{,}2 \cdot \cos 60 = 0$

$\circlearrowleft A \quad R_B \cdot 6 - 0{,}5 \cdot 9 \cdot 4{,}5 - 1{,}2 \cdot \sin 60 \cdot 9 = 0$

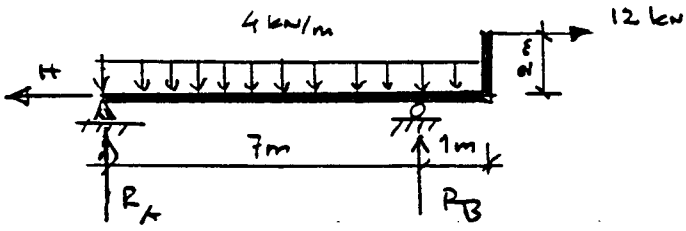
$\uparrow \quad R_A + R_B - 0{,}5 \cdot 9 - 1{,}2 \cdot \sin 60 = 0$

$H = 1{,}6 \text{ kN}$ (åt höger)

$R_B = 4{,}93 \text{ kN}$

$R_A = 0{,}61 \text{ kN}$

Ex 22



$\rightarrow \quad 12 - H = 0$

$\circlearrowleft A \quad R_B \cdot 7 - 4 \cdot 8 \cdot 4 - 12 \cdot 2 = 0$

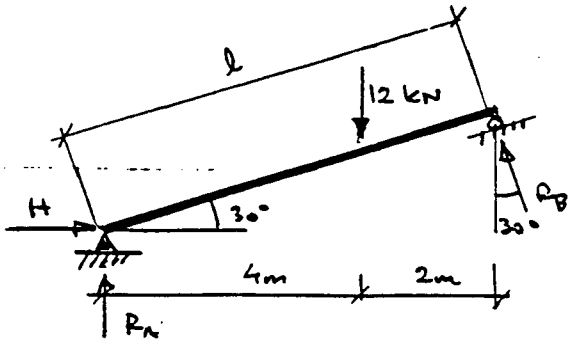
$\uparrow \quad R_A + R_B - 4 \cdot 8 = 0$

$H = 12 \text{ kN}$ (åt vänster)

$R_B = 21{,}7 \text{ kN}$

$R_A = 10{,}3 \text{ kN}$

Ex 23



$$l = \frac{6}{\cos 30} = 6{,}93\,m$$

$\circlearrowleft A$ $\quad R_B \cdot 6{,}93 - 12 \cdot 4 = 0$

$\uparrow$ $\quad R_A - 12 + R_B \cdot \cos 30 = 0$

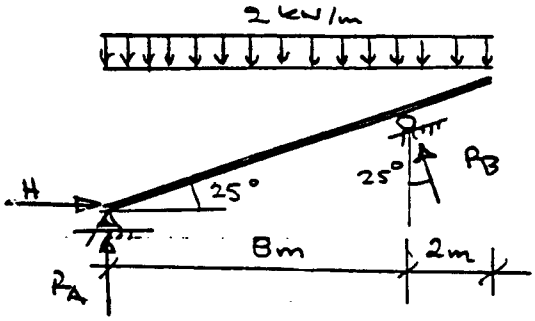
$\rightarrow$ $\quad H - R_B \cdot \sin 30 = 0$

$R_B = 6{,}93$ kN

$R_A = 6{,}0$ kN

$H = 3{,}46$ kN (åt höger)

Ex 24



$\circlearrowleft A$ $\quad R_B \cdot \frac{8}{\cos 25} - 2 \cdot 10 \cdot 5 = 0$

$\uparrow$ $\quad R_A + R_B \cdot \cos 25 - 2 \cdot 10 = 0$

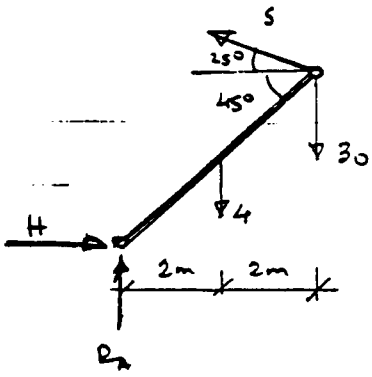
$\rightarrow$ $\quad H - R_B \cdot \sin 25 = 0$

$R_B = 11{,}3$ kN

$R_A = 9{,}7$ kN

$H = 4{,}8$ kN (åt höger)

Ex 25



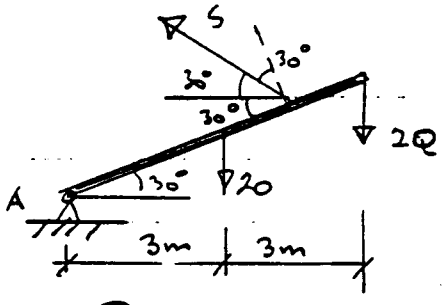
$$\circlearrowleft A \quad 4\cdot 2 + 30\cdot 4 - S\sin 25\cdot 4 - S\cos 25\cdot 4 = 0$$

$$\underline{S = 24{,}1 \text{ kN}}$$

$$H = 24{,}1\cdot\cos 25 = \underline{21{,}8 \text{ kN}}$$

$$R_A = 34 - 24{,}1\cdot\sin 25 = \underline{23{,}8 \text{ kN}}$$

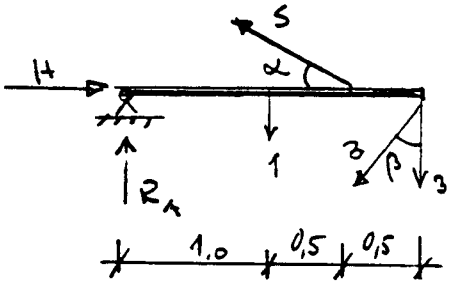
Ex 26



$$\circlearrowleft A \quad 20\cdot 3 + 2Q\cdot 6 - 105\cancel{\cos 30}\cdot\frac{4}{\cancel{\cos 30}} = 0$$

$$\underline{Q = 30 \text{ kN}}$$

Ex 27



$$\tan\alpha = \frac{1}{1{,}5} \qquad \alpha = 33{,}7°$$

$$\tan\beta = \frac{2}{3} \qquad \beta = 33{,}7°$$

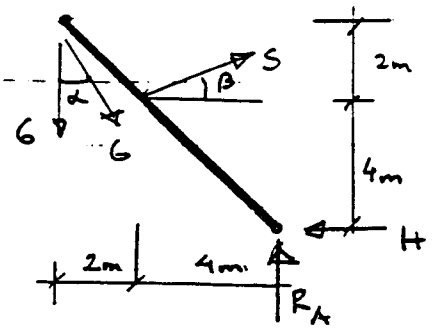
$$\circlearrowleft A \quad 1\cdot 1 + (3 + 3\cos 33{,}7)\cdot 2 - S\sin 33{,}7\cdot 1{,}5 = 0$$

$$\underline{S = 14{,}4 \text{ kN}}$$

$$H = 14{,}4\cdot\cos 33{,}7 + 3\sin 33{,}7 = \underline{13{,}6 \text{ kN}}$$

$$R_A = (3 + 3\cdot\cos 33{,}7) + 1 - 14{,}4\cdot\sin 33{,}7 = \underline{-1{,}5 \text{ kN}}$$

Ex 28



$\tan\alpha = \frac{6}{10}$ $\quad \alpha = 31°$

$\tan\beta = \frac{2}{4}$ $\quad \beta = 26{,}56°$

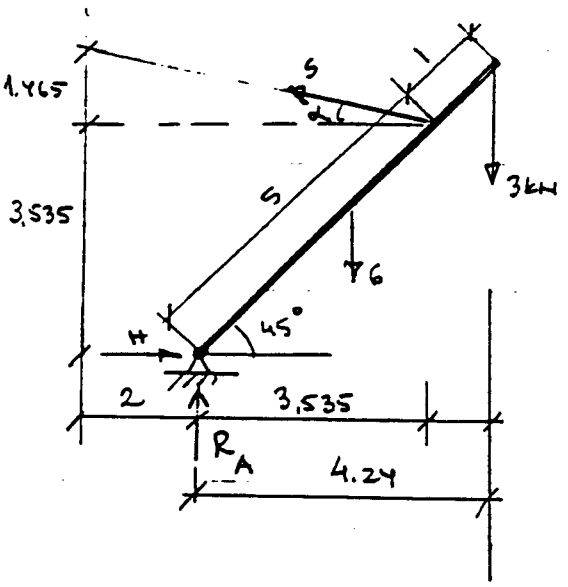
$\circlearrowleft A$ $\quad S\cdot\cos 26{,}56\cdot 4 + S\sin 26{,}56\cdot 4 - (6+6\cos 31)\cdot 6$
$+ 6\sin 31\cdot 6 = 0$

$S = \frac{48{,}3}{5{,}366} = \underline{9{,}0 \text{ kN}}$

$H = 6\sin 31 + 9\cdot\cos 26{,}56 = \underline{11{,}1 \text{ kN}}$ (åt vänster)

$R_A = 6 + 6\cos 31 - 9\cdot\sin 26{,}56 = \underline{7{,}1 \text{ kN}}$

Ex 29



$\tan\alpha = \frac{1.465}{5{,}535}$ $\quad \underline{\alpha = 14{,}8°}$

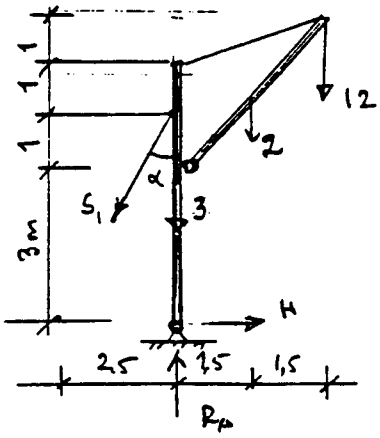
$\circlearrowleft A$ $\quad 6\cdot 2{,}12 + 3\cdot 4.24 - S\cdot\cos 30{,}2\cdot 5 =$

$\underline{S = 5{,}9 \text{ kN}}$

$H = 5{,}89\cdot\cos 14{,}8 = \underline{5{,}7 \text{ kN}}$ (åt höger)

$R_A = 6 + 3 - 5{,}89\cdot\sin 14{,}8 = \underline{7{,}5 \text{ kN}}$

a)



$\tan\alpha = \frac{3,5}{4}$ $\alpha = 32,0°$

Ex 30

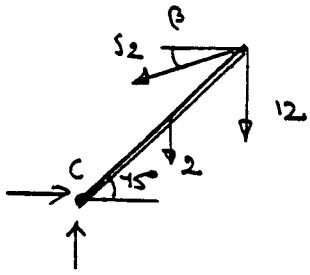
$\circlearrowleft A$ $2\cdot 1,5 + 12\cdot 3 - S_1 \sin 32 \cdot 4 = 0$

$S_1 = 18,4$ kN

$H_A = 18,4 \cdot \sin 32 = 9,75$ kN (åt höger)

$R_A = 2 + 3 + 12 + 18,4 \cdot \cos 32 = 32,6$ kN

b)

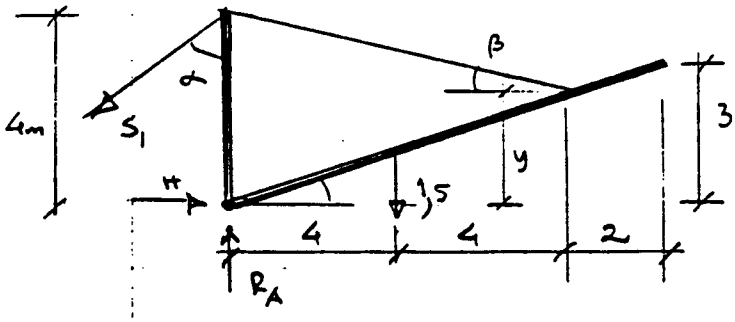


$\tan\beta = \frac{1}{3}$ $\beta = 18,4°$

$\circlearrowleft C$ $2\cdot 1,5 + 12\cdot 3 - S_2 \cdot \cos 18,4 \cdot 3 +$
$+ S_2 \sin 18,4 \cdot 3 = 0$

$S_2 = 20,5$ kN

Ex 31



a) $\tan\alpha = \frac{5}{4}$ $\alpha = 51,3°$

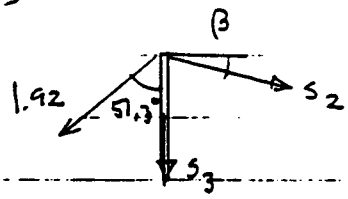
$\circlearrowleft A$ $1,5\cdot 4 - S_1 \sin 51,3 \cdot 4 = 0$

$S_1 = 1,92$ kN

$H = 1,92 \sin 51,3 = 1,50$ kN

$R_A = 1,5 + 1,92 \cdot \cos 51,3 = 2,7$ kN

b) Likformighet ger $\frac{y}{8} = \frac{3}{10}$ $y = 2,4$ m

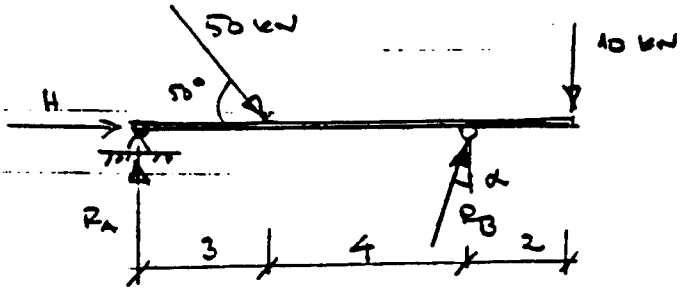


$\tan\beta = \frac{4-2,4}{8}$ $\beta = 11,3°$

$\leftarrow$ $1,92 \cdot \sin 51,3 - S_2 \cdot \cos 11,3 = 0$ $S_2 = 1,53$ kN

$S_3 = -1,92 \cdot \cos 51,3 - 1,53 \cdot \sin 11,3$ $S_3 = -1,50$ kN

Ex 32



$\tan\alpha = \frac{2}{5} \qquad \alpha = 21{,}8°$

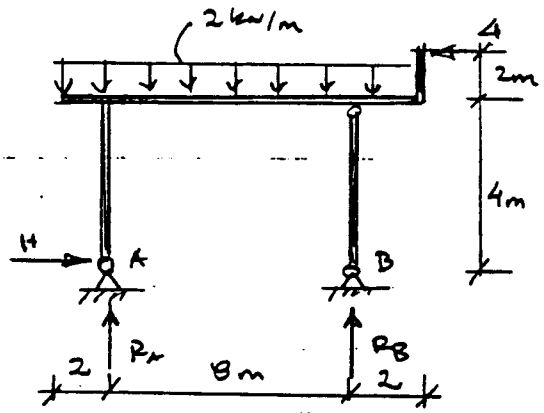
$\circlearrowleft A \quad 50 \cdot \sin 50 \cdot 3 + 10 \cdot 9 - R_B \cdot \cos 21{,}8 \cdot 7 = 0$

$\underline{R_B = 31{,}5 \text{ kN}}$

$H = -50 \cdot \cos 50 - 31{,}5 \cdot \sin 21{,}8 = \underline{-43{,}8 \text{ kN}}$ (åt vänster)

$R_A = 50 \cdot \sin 50 + 10 - 31{,}5 \cdot \cos 21{,}8 = \underline{19{,}0 \text{ kN}}$

Ex 33

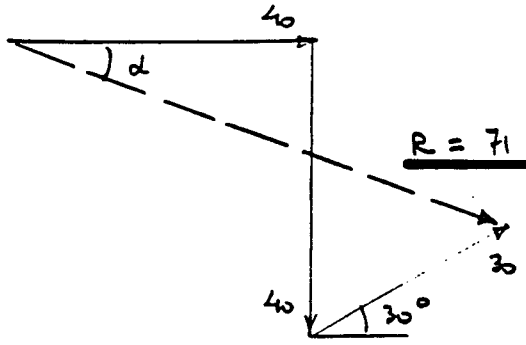


$\circlearrowleft A \quad 2 \cdot 12 \cdot 4 - 4 \cdot 6 - R_B \cdot 8 = 0$

$\underline{R_B = 9 \text{ kN}}$

$\underline{H = 4 \text{ kN}} \qquad R_A = 2 \cdot 12 - 9 = \underline{15 \text{ kN}}$

Ex 34



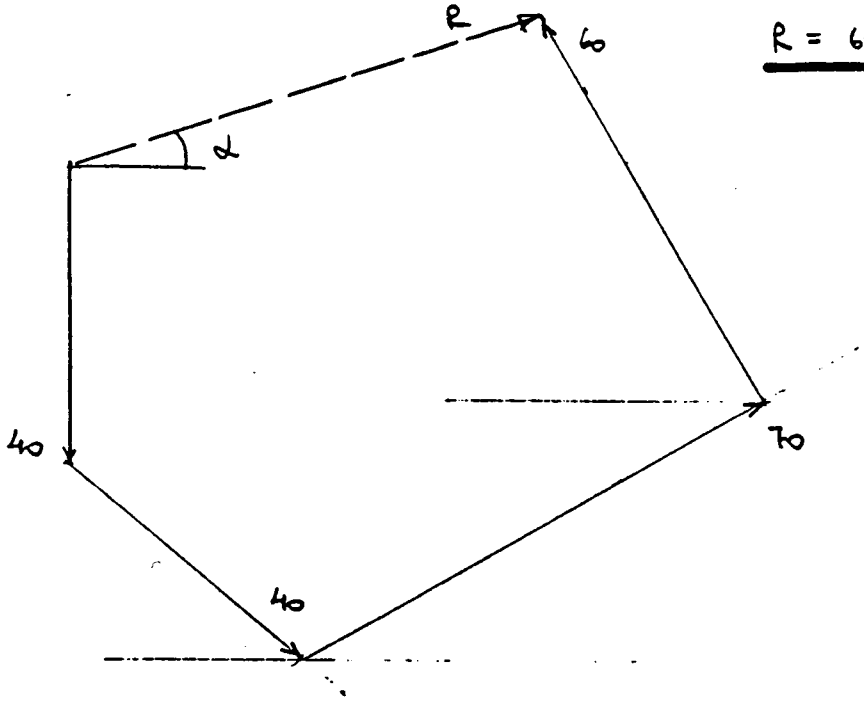
R = 71 kN ; lutning 21° mot höger

Analytisk kontroll

$R_x = 40 + 30 \cdot \cos 30 = 66{,}0$ $\qquad \tan\alpha = \frac{25}{66}$ $\qquad \alpha = 21°$

$R_y = 40 - 30 \cdot \sin 30 = 25{,}0$

$R = \sqrt{66^2 + 25^2} = 70{,}58 \sim 71$ kN



R = 65 kN ; lutning 19°

upåt höger

Ex 35

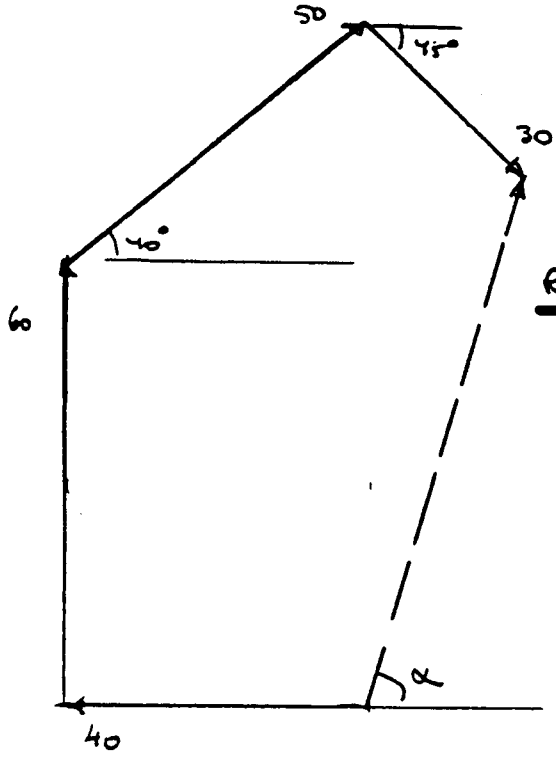
Analytisk kontroll

$R_x = 40 \cos 40 + 70 \cos 30 - 60 \cdot \cos 60 = 61{,}3$

$R_y = 40 \cdot \sin 40 + 40 - 70 \cdot \sin 30 - 60 \cdot \sin 60 = -21{,}25$

$R = \sqrt{61{,}3^2 + 21{,}25^2} = 65$ kN

$\tan\alpha = \frac{21.25}{43} = 19°$



Ex 36

R = 74 kN i lutning 75° mot høyre

Analytisk kontroll

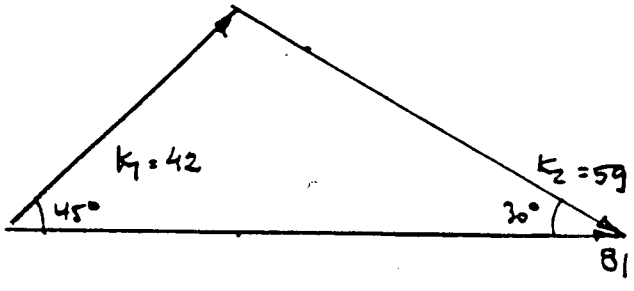
$R_x = -40 + 50 \cdot \cos 40 + 30 \cdot \cos 45 = 19{,}5 \text{ kN}$

$R_y = 60 + 50 \cdot \sin 40 - 30 \cdot \sin 45 = 70{,}9$

$R = \sqrt{19.5^2 + \cdot 70.9^2} = 73{,}6 \sim \underline{74 \text{ kN}}$

$\tan \alpha = \frac{70.9}{19.5} = \underline{75°}$

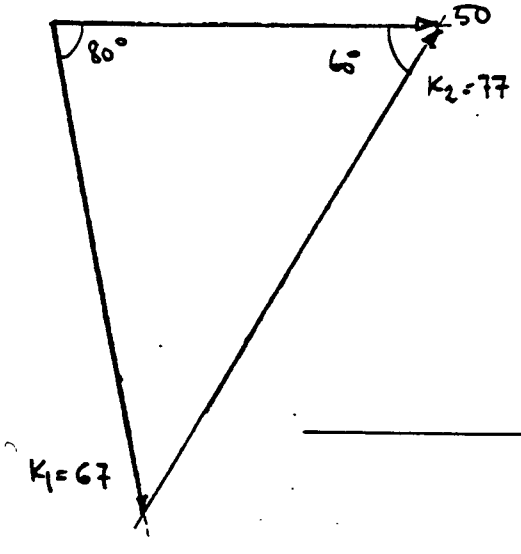
Analytisk kontroll: Ex 37



$$\begin{cases} K_1 \cos 45 + K_2 \cos 30 = 81 \\ K_1 \sin 45 - K_2 \sin 30 = 0 \end{cases} \quad K_2 = 1{,}414 \cdot K_1$$

$K_1 (\cos 45 + 1{,}414 \cdot \cos 30) = 81 \quad \underline{K_1 = 42}$

$K_2 = 1{,}414 \cdot 42 = \underline{59 \text{ kN}}$



Ex 38

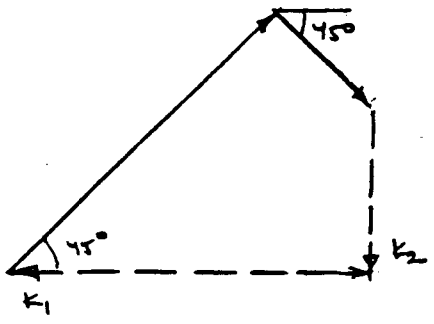
$K_1 \cos 80 + K_2 \cos 60 = 50$

$K_1 \sin 80 - K_2 \cdot \sin 60 = 0 \quad K_2 = 1{,}137 \cdot K_1$

$K_1 (\cos 80 + 1{,}137 \cdot \cos 60) = 50$

$\underline{K_1 = 67{,}4} \quad K_2 = 1{,}137 \cdot 67.4 = \underline{77 \text{ kN}}$

Ex 39



Avläsning ger $K_1 = 48$ kN åt vänster

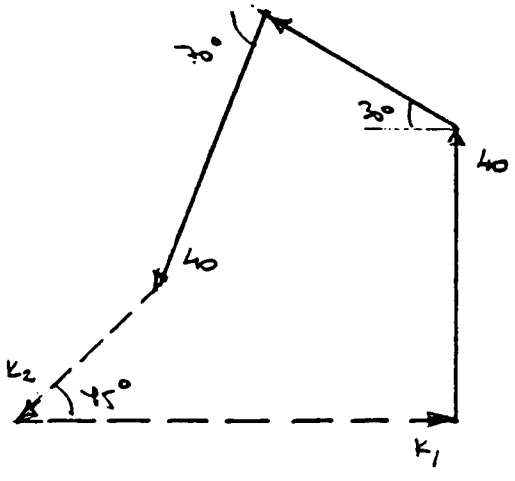
$K_2 = 22$ kN nedåt

Analytisk kontroll:

$K_1 = 50 \cdot \cos 45 + 21 \cdot \cos 45 = 48$ kN

$K_2 = 50 \cdot \sin 45 - 21 \sin 45 = 22$ kN

Ex 40



Avläsning ger

$K_1 = 57$ kN åt höger

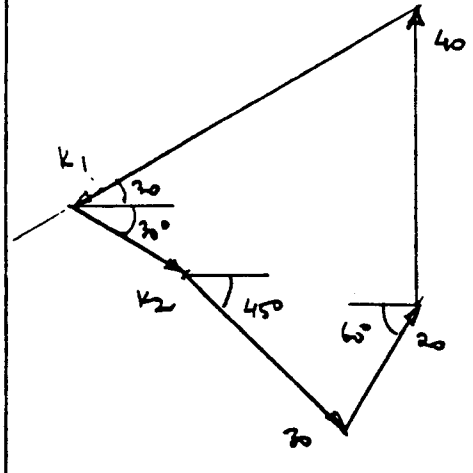
$K_2 = 25$ kN mot nedåt vänster

Analytisk kontroll:

$$K_2 = \frac{40 + 30 \cdot \sin 30 - 40 \cdot \sin 30}{\sin 45} = 24{,}6$$

$$K_1 = 30 \cdot \cos 30 + 40 \cdot \cos 30 + 24{,}6 \cdot \cos 45 = 57$$

Ex 41



Avläsning ger

$K_1 = 54$ kN mot nedåt vänster

$K_2 = 18$ kN mot nedåt höger

Analytisk kontroll:

$\downarrow$ $K_1 \cdot \sin 30 + K_2 \cdot \sin 30 + 30 \sin 45 - 20 \sin 60 - 40 = 0$

$$\boxed{K_1 + K_2 = 72{,}2}$$

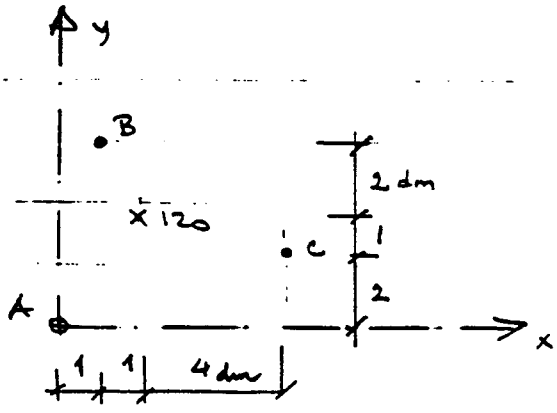
$\rightarrow$ $K_2 \cdot \cos 30 - K_1 \cdot \cos 30 + 30 \cos 45 + 20 \cdot \cos 60 = 0$

$$\boxed{K_1 - K_2 = 36{,}0}$$

$2K_1 = 108$

$K_1 = 54$ kN $K_2 = 18$ kN

Ex 42



$\uparrow \; A + B + C - 120 = 0$

$\curvearrowright_A \; 120 \cdot 2 - B \cdot 1 - C \cdot 6 = 0$

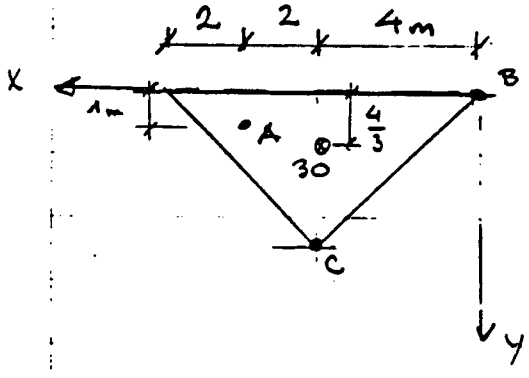
$\curvearrowright_B \; 120 \cdot 3 - B \cdot 5 - C \cdot 2 = 0$

Den andre gir $B = 240 - 6C$

$$\begin{cases} A + 240 - 6C + C - 120 = 0 \\ 360 - 1200 + 30C - 2C = 0 \end{cases} \Rightarrow C = \frac{840}{28} = \underline{30\,kN}$$

$B = 240 - 6 \cdot 30 = \underline{60\,kN}$ $\quad A = 120 - 30 - 60 = \underline{30\,kN}$

Ex 43



$\uparrow \; A + B + C - 30 = 0$

$\curvearrowright_x \; 30 \cdot \frac{4}{3} - C \cdot 4 - A \cdot 1 = 0$

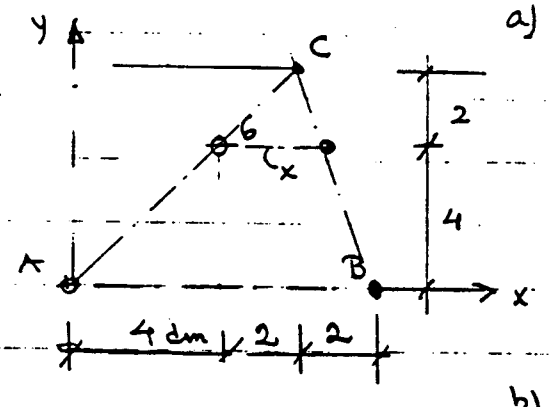
$\curvearrowright_y \; 30 \cdot 4 - C \cdot 4 - A \cdot 6 = 0$

Andre ekv. gir $A = 40 - 4C$

$$\begin{cases} 40 - 4C + B + C - 30 = 0 \\ 120 - 4C - 240 + 24C = 0 \end{cases} \Rightarrow C = \frac{120}{20} = \underline{6\,kN}$$

$A = 40 - 24 = \underline{16\,kN}$ $\quad B = 30 - 16 - 6 = \underline{8\,kN}$

Ex 44



a)

$\uparrow \; A + B + C - 6 = 0$

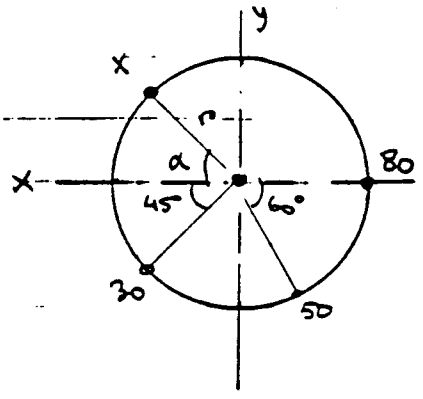
$\curvearrowright_x \; 6 \cdot 4 - C \cdot 6 = 0$ $\quad \underline{C = 4\,kN}$

$\curvearrowright_y \; 6 \cdot 4 - C \cdot 6 - B \cdot 8 = 0$ $\quad \underline{B = 0}$

$\underline{A = 2\,kN}$

b) $\frac{x}{8} = \frac{2}{6}$ $\quad x = \frac{16}{6} = \underline{2{,}67\,dm}$

Ex 45



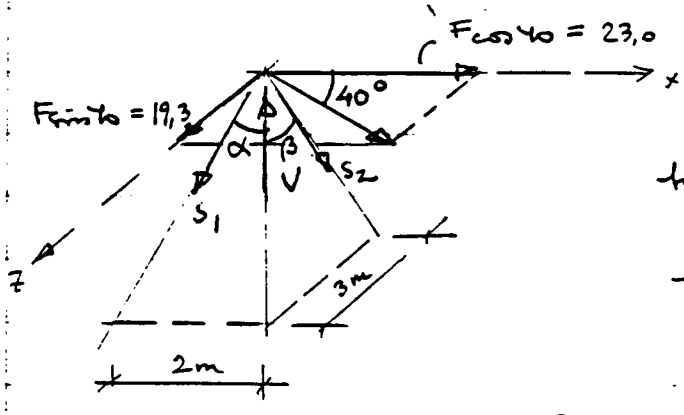
$\circlearrowleft x$ $\quad x \cdot \cancel{r} \cdot \sin\alpha - 30 \cdot \cancel{r} \sin 45 - 50 \cdot \cancel{r} \cdot \sin 60 = 0$

$\circlearrowleft y$ $\quad 80 \cdot \cancel{r} + 50 \cdot \cancel{r} \cdot \cos 60 - 30 \cdot \cancel{r} \cos 45 - x \cdot \cancel{r} \cos\alpha = 0$

$$\begin{cases} x \cdot \sin\alpha = 64{,}5 \\ x \cdot \cos\alpha = 83{,}8 \end{cases}$$

$\tan\alpha = 0{,}770 \qquad \underline{\alpha = 37{,}6°}$

Ex 46



$F = 30$ kN

$\tan\alpha = \frac{2}{5} \qquad \alpha = 21{,}8°$

$\tan\beta = \frac{3}{5} \qquad \beta = 31°$

$\rightarrow x \quad 23{,}0 - S_1 \cdot \sin 21{,}8 = 0$

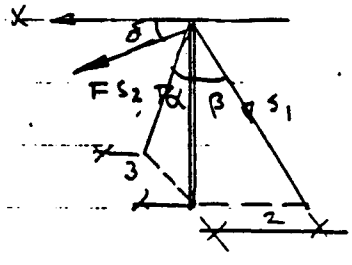
$\swarrow z \quad 19{,}3 - S_2 \cdot \sin 31 = 0$

$y \uparrow \quad V - S_1 \cdot \cos 21{,}8 - S_2 \cos 31 = 0$

$S_1 = \underline{61{,}9 \text{ kN}} \qquad S_2 = \underline{37{,}5 \text{ kN}}$

$V = 61{,}9 \cdot \cos 21{,}8 + 37{,}5 \cdot \cos 31 = \underline{89{,}6 \text{ kN}}$

Ex 47



$\tan\alpha = \frac{3}{3} \qquad \alpha = 45°$

$\tan\beta = \frac{2}{3} \qquad \beta = 33{,}7°$

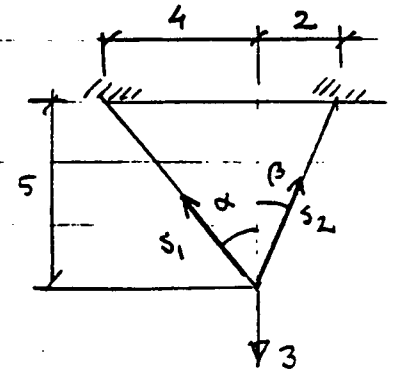
$F \cdot \cos\delta - S_1 \cdot \sin 33{,}7° = 0$

$F \cdot \sin\delta - S_2 \cdot \sin 45° = 0$

$S_1 = S_2$ ger $\quad \frac{\sin\delta}{\cos\delta} = \frac{\sin 45}{\sin 33{,}7}$

$\underline{\delta = 51{,}9°}$

Ex 48



$\tan\alpha = \frac{4}{5} \quad \alpha = 38{,}66°$

$\tan\beta = \frac{2}{5} \quad \beta = 21{,}8°$

$\uparrow \; S_1 \cdot \cos 38{,}66 + S_2 \cdot \cos 21{,}8 - 3 = 0$

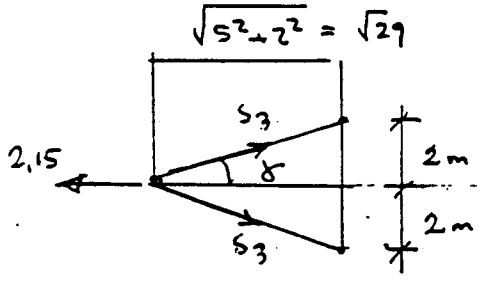
$\rightarrow \; S_1 \sin 38{,}66 - S_2 \sin 21{,}8 = 0$

Den andra ekv ger $\quad S_2 = 1{,}682 \cdot S_1$

$S_1 \left(\cos 38{,}66 + 1.682 \cdot \cos 21.8 \right) = 3$

$S_1 = \frac{3}{2{,}34} = \underline{1{,}28 \text{ kN}}$

$S_2 = 1{,}682 \cdot 1{,}28 = \underline{2{,}15 \text{ kN}}$

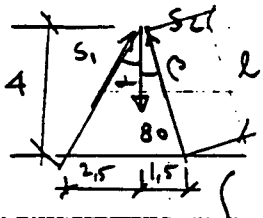


$\tan\gamma = \frac{2}{\sqrt{29}} \quad \gamma = 20{,}4°$

$2{,}15 - 2 \cdot S_3 \cdot \cos 20{,}4 = 0$

$\underline{S_3 = 1{,}15 \text{ kN}}$

Ex 49



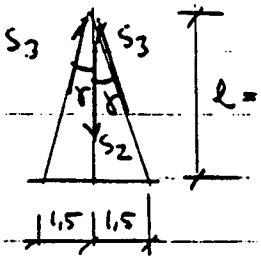
$\tan\alpha = \frac{2{,}5}{4} \quad \alpha = 32{,}0°$

$\tan\beta = \frac{1{,}5}{4} \quad \alpha = 20{,}56°$

$\uparrow \; S_1 \cos 32 + S_2 \cos 20{,}56 - 80 = 0$

$\rightarrow \; S_1 \sin 32 - S_2 \sin 20{,}56 = 0$

$S_2 = 1{,}503 \, S_1$ ... ärnett ger $\underline{S_1 = 35{,}4 \text{ kN}}$ $\quad S_2 = 53{,}4$

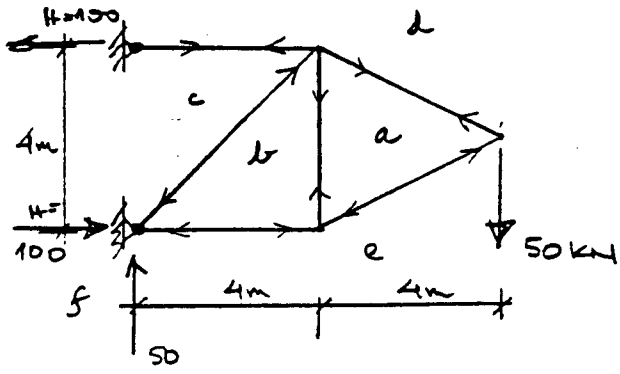


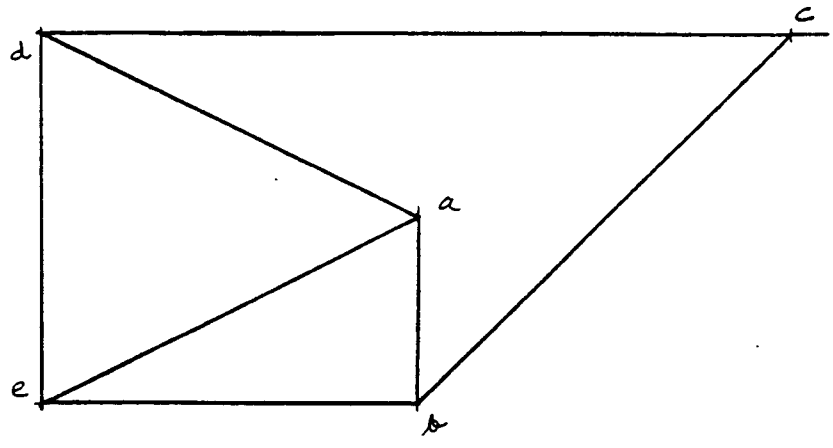
$l = \sqrt{4^2 + 1{,}5^2} = 4{,}272 \text{ m} \qquad \tan\gamma = \frac{1{,}5}{4{,}272} \qquad \gamma = 19{,}35°$

$\uparrow \; 2 \cdot S_3 \cdot \cos 19{,}35 - S_2 = 0$

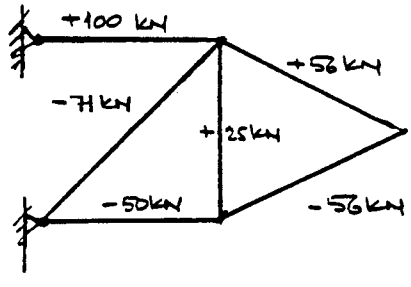
$S_3 = \frac{53{,}4}{2 \cdot \cos 19{,}35} = \underline{28{,}3 \text{ kN}}$

Ex 4:2


H=100
d
c
4m
b
a
H=
100
e
50 kN
f
4m
4m
50

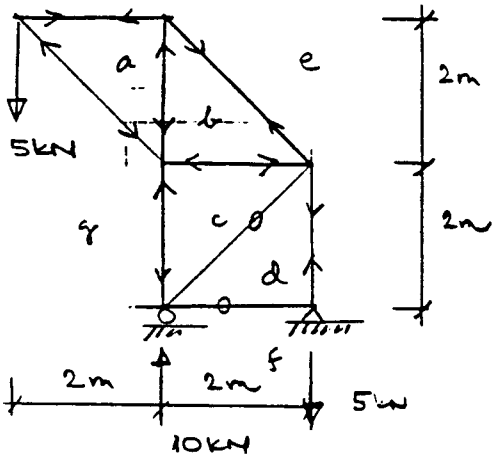

d
c
a
e
b

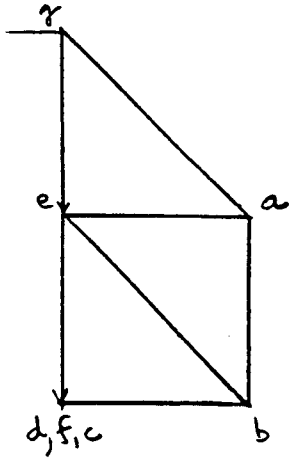
Svar:

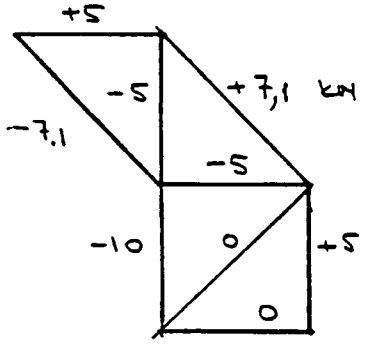

+100 kN
+56 kN
-71 kN
+25 kN
-50 kN
-56 kN

Ex 4:3

Ex 52

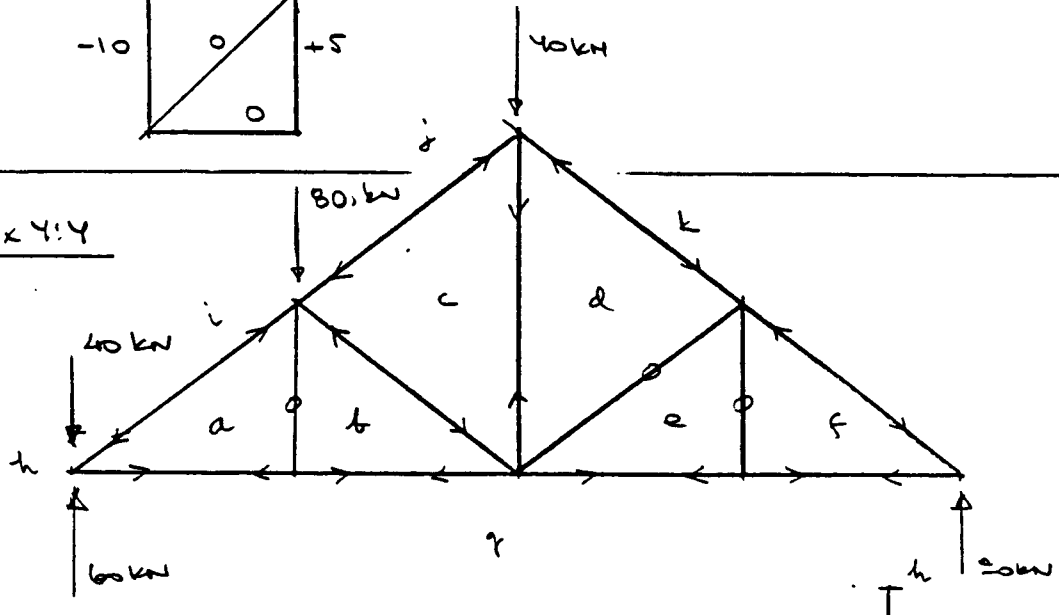


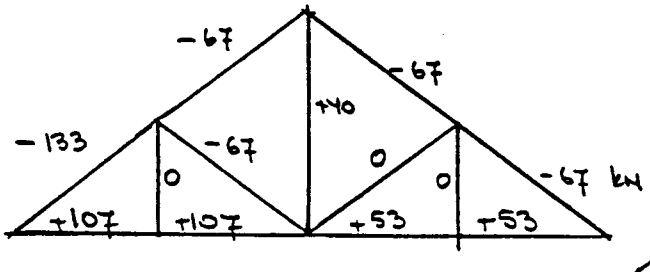


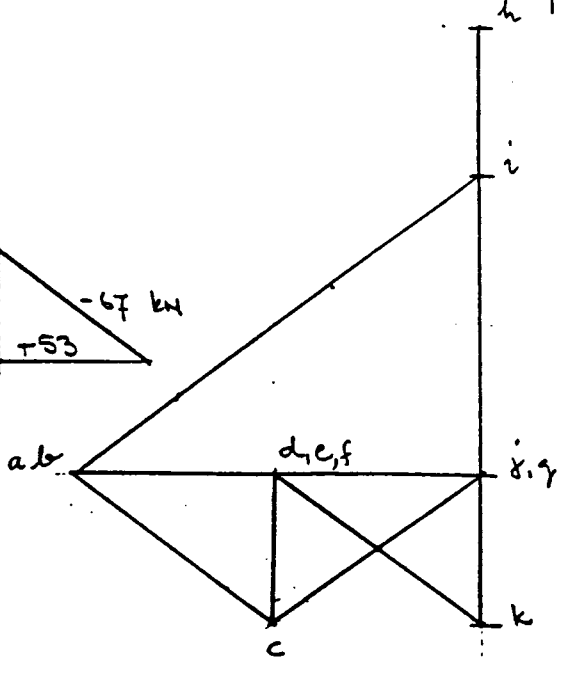


Ex 4:4

Ex 53

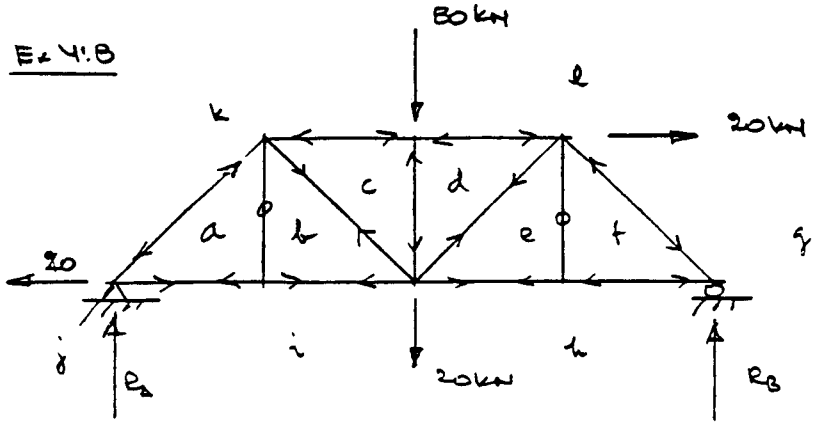






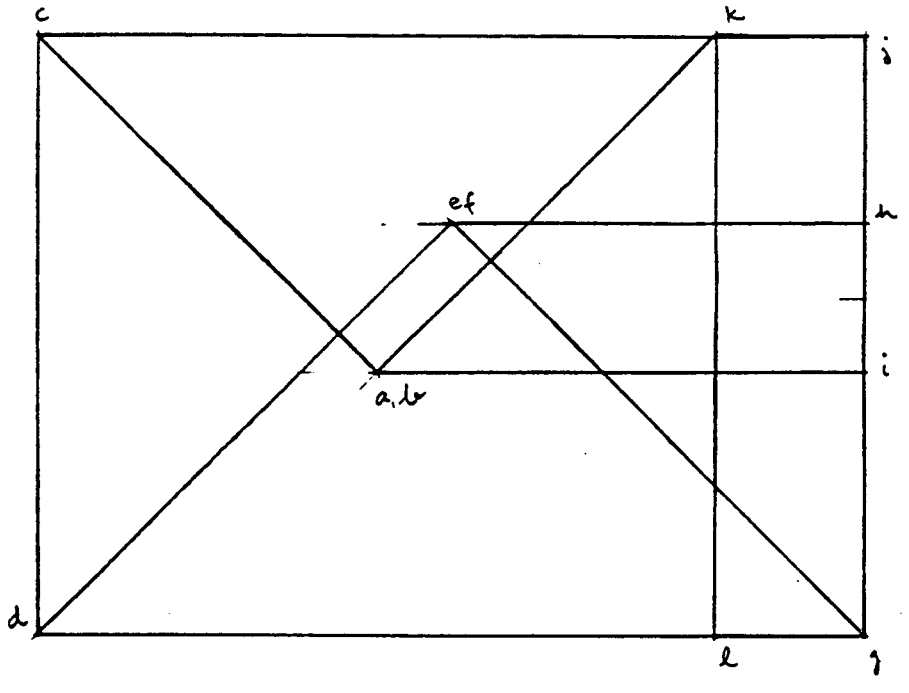
Ex 4.8

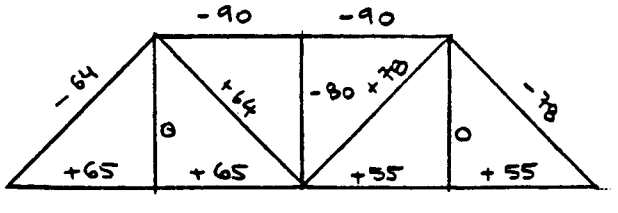
Ex 54



$\circlearrowleft_A$ $80 \cdot 2 + 20 \cdot 2 + 20 \cdot 1 - R_B \cdot 4 = 0$ $R_B = 55$

$R_A = 45$

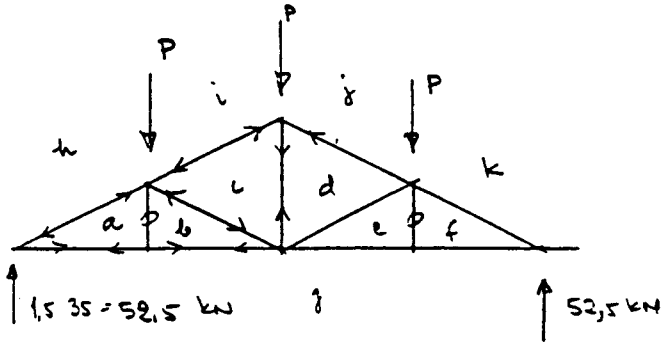


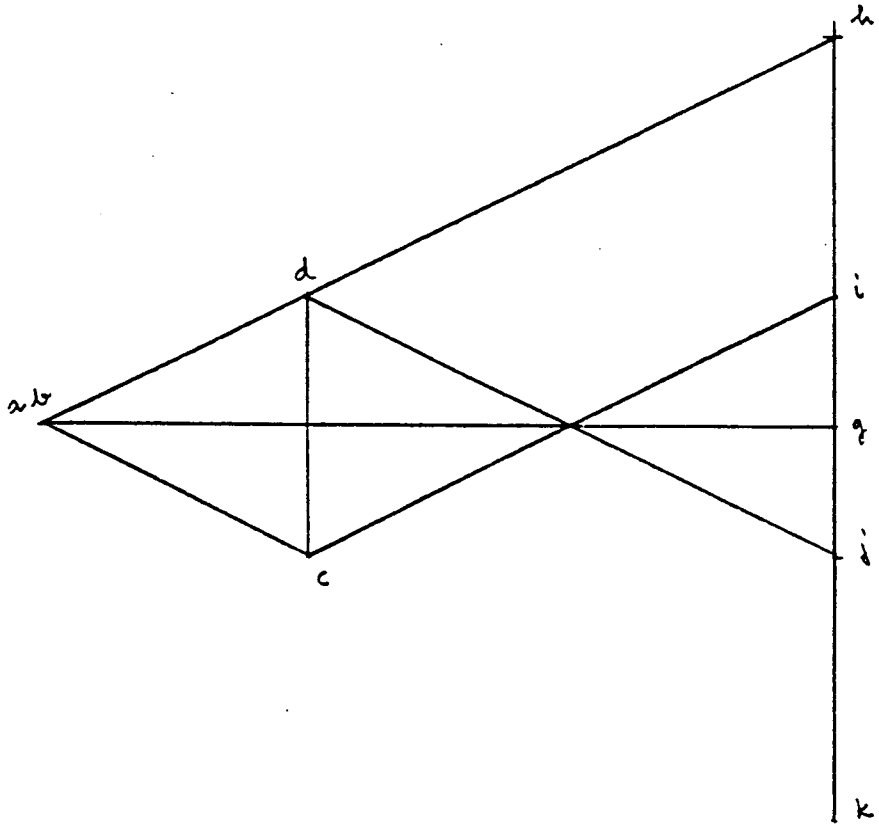


Svar

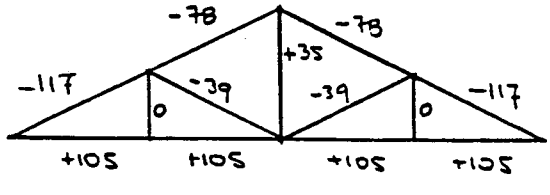
Ex 4:6

$P = 10 \cdot 3{,}5 = 35$ kN



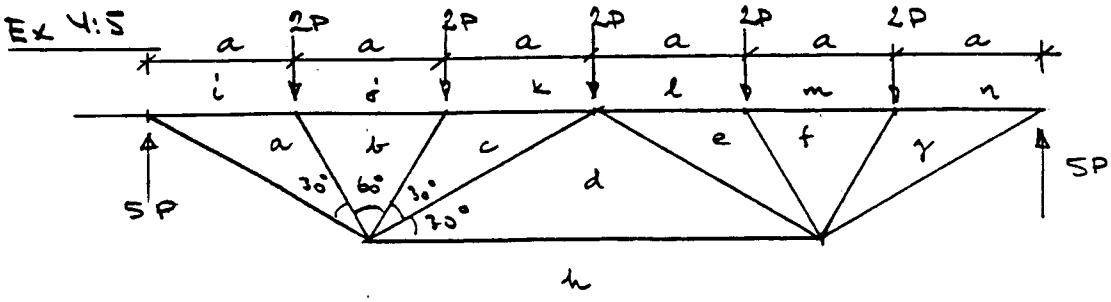


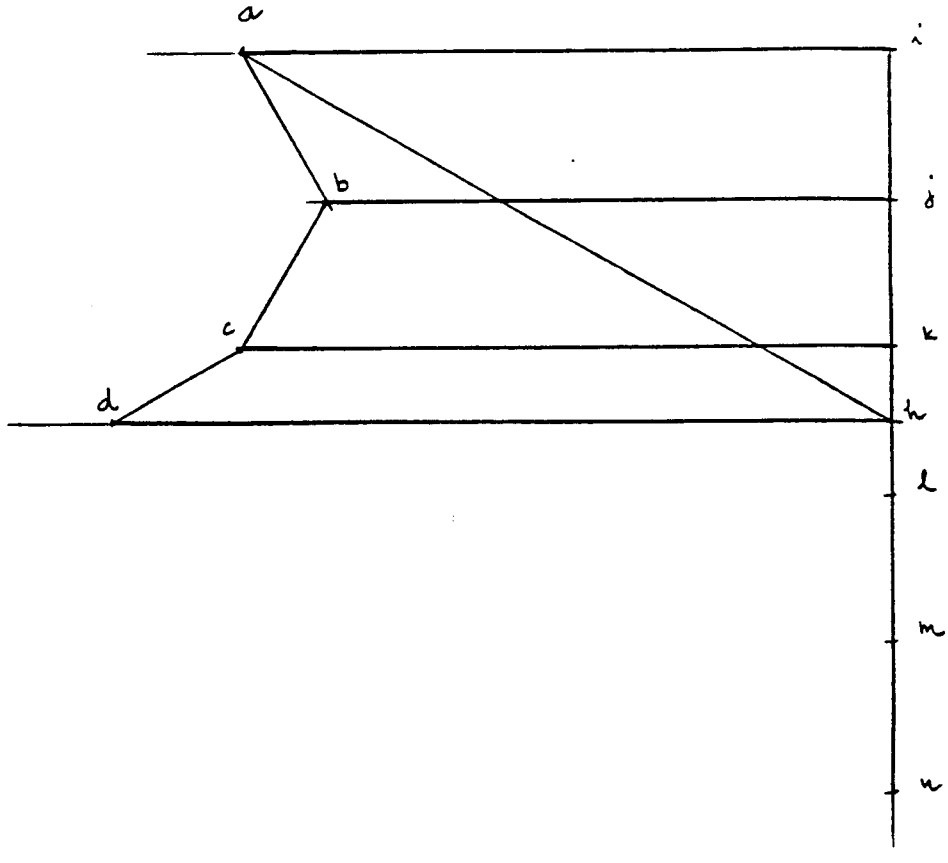
Svar :



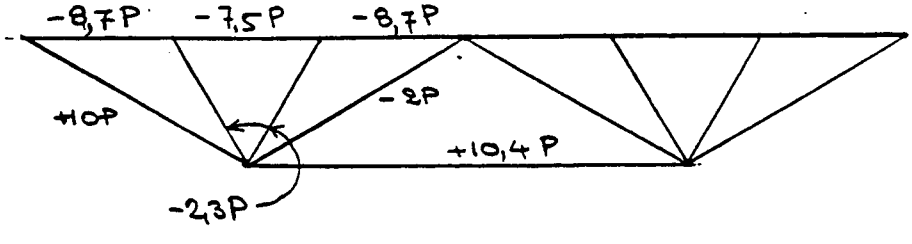
Ex 4:5

Ex 56



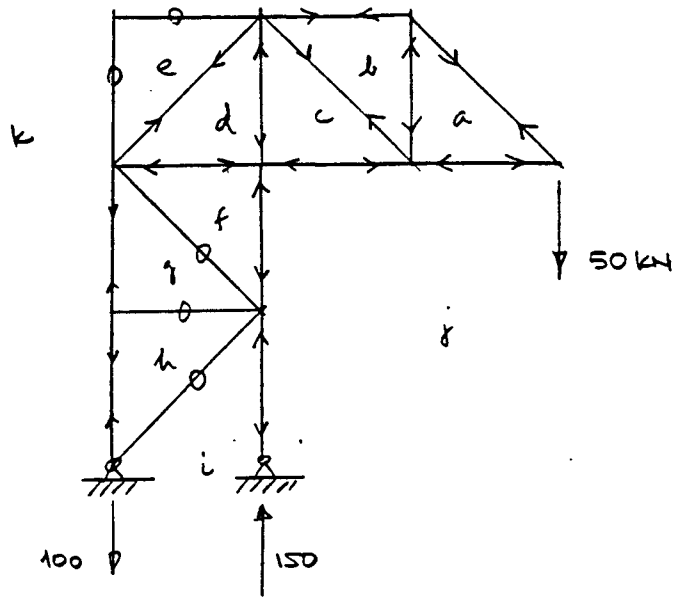


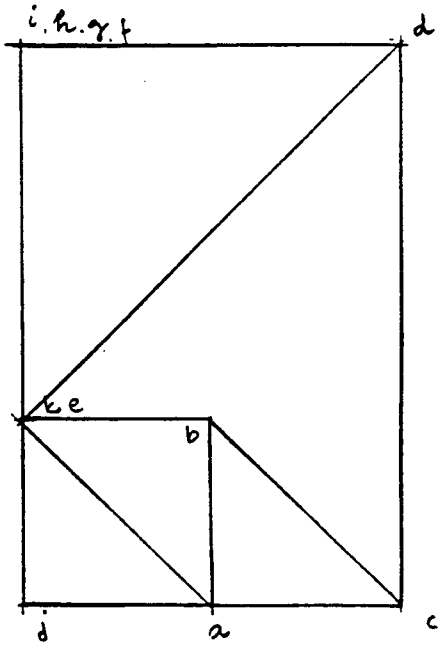
SVAR

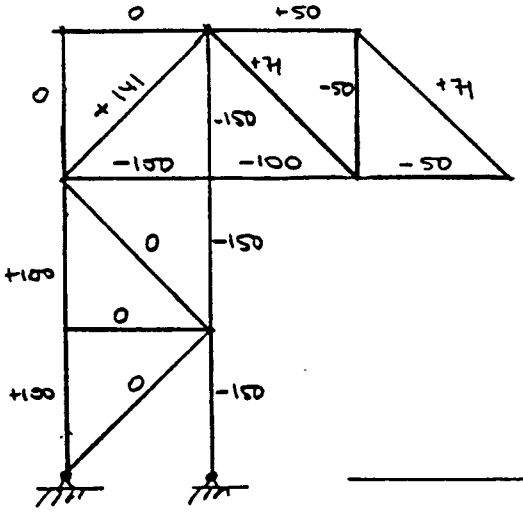


Ex 4:9

Ex 57

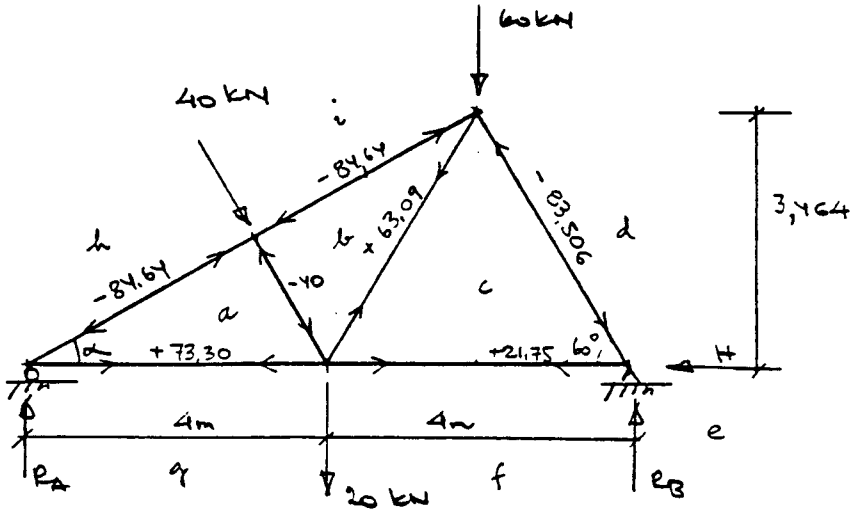

e
b
d
c
a
k
f
g
h
i
j
50 kN
100
150


i.h.g.f
d
k e
b
j
a
c


0
+50
0
+141
+71
-50
+71
-150
-150
-100
-50
0
-150
+100
0
+100
0
-150

Ex 4:7

Ex 5B



$\sin\alpha = \frac{4}{8}$ $\alpha = 30°$

$\leftarrow$ $H - 40\sin 30 = 0$ $H = 20$ kN

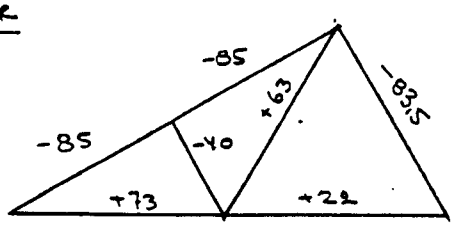
$\circlearrowright A$ $40 \cdot 4 \cdot \cos 30 + 60 \cdot 6 - R_B \cdot 8 + 20 \cdot 4 = 0$

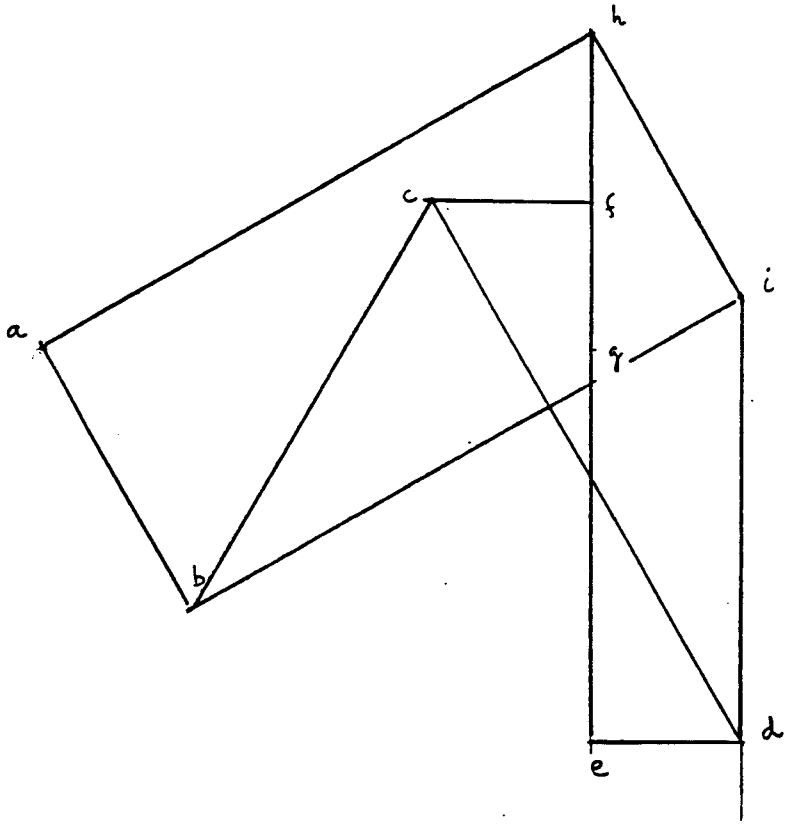
$R_B = 72{,}32$ kN

$\uparrow$ $R_A + 72{,}3 - 20 - 60 - 40\cos 30 = 0$

$R_A = 42{,}32$ kN

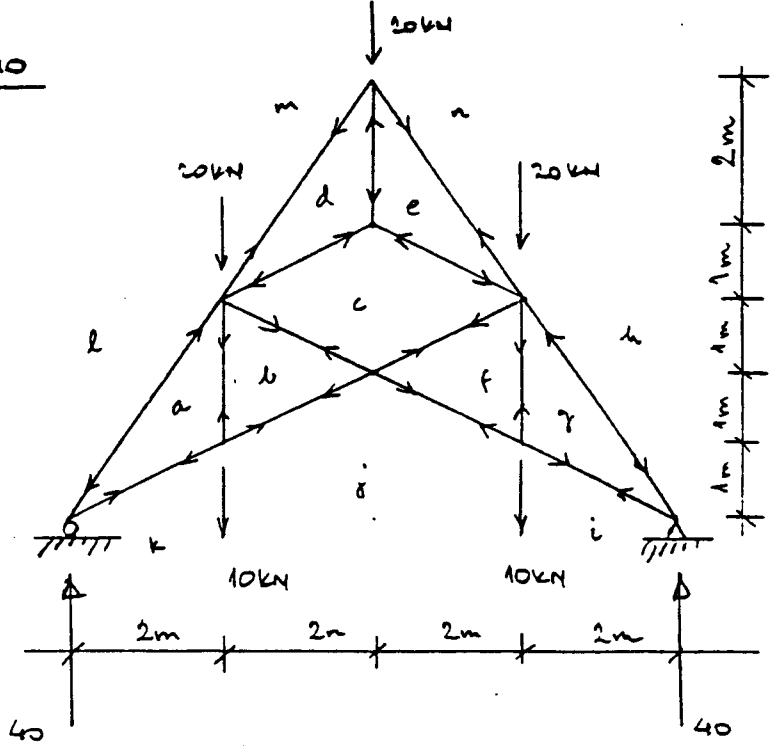
Svar

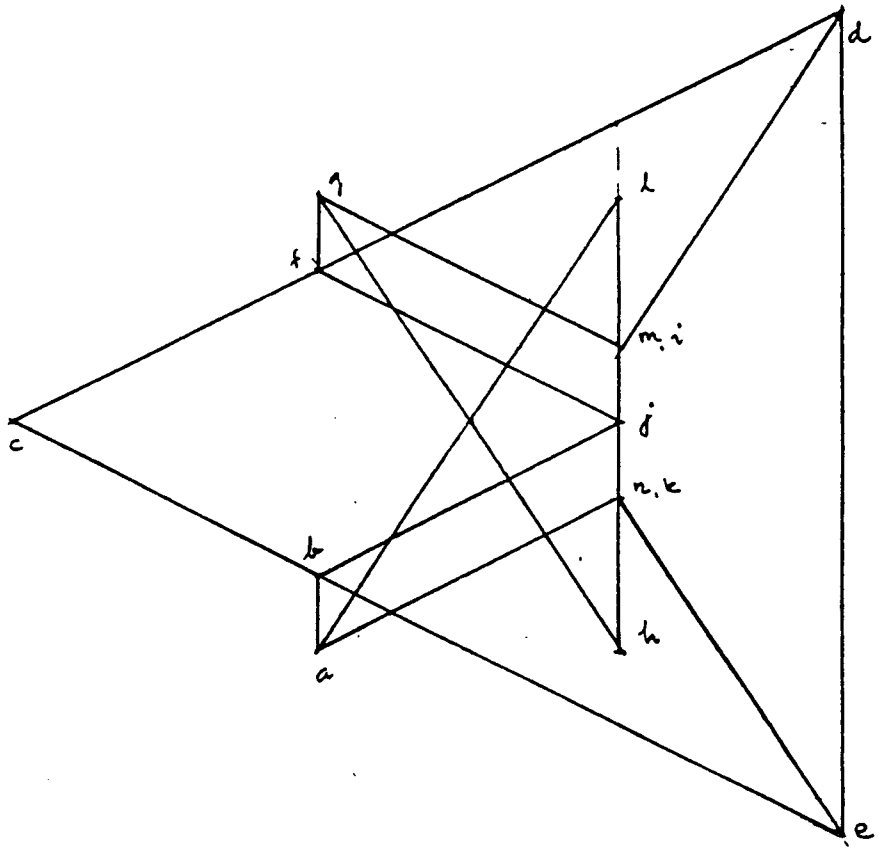


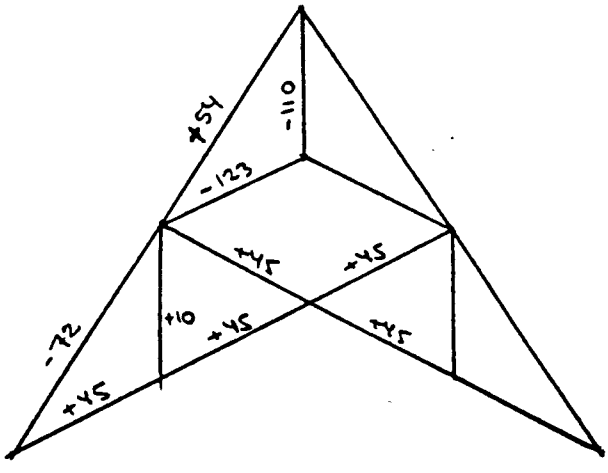


Ex 4:10

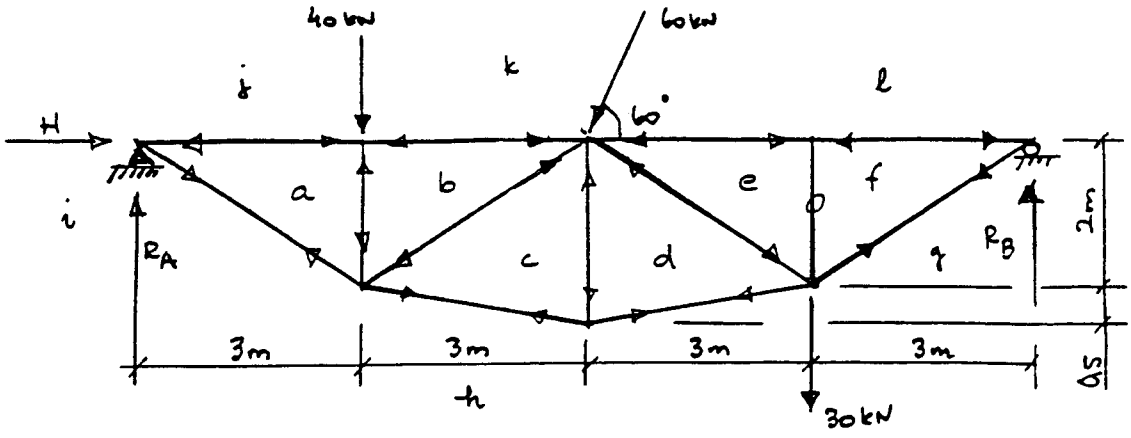
Ex 59


20kN
20kN
20kN
m
n
d
e
c
l
h
a
b
f
g
j
k
i
10kN
10kN
2m
2m
2m
2m
2m
1m
1m
1m
1m
2m
40
40


d
g
l
f
m, i
c
j
n, k
b
h
a
e


+54
-110
-123
+45
+45
-72
+10
+45
+45
+45

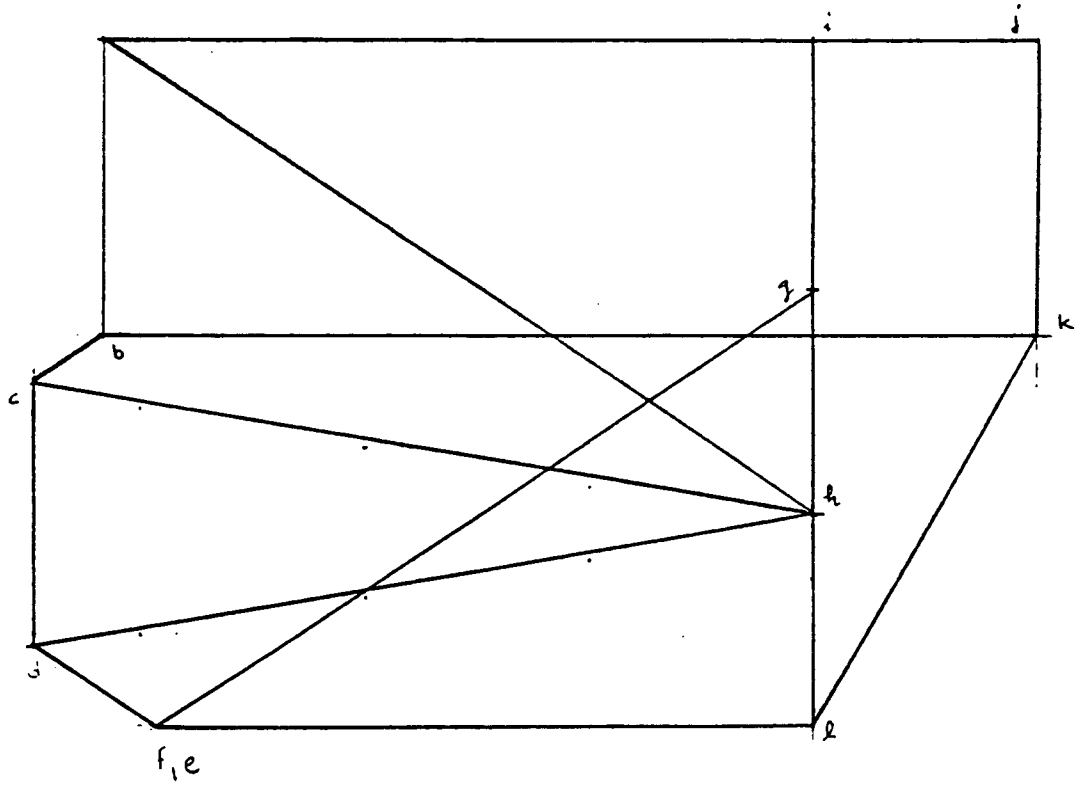
Ex 60

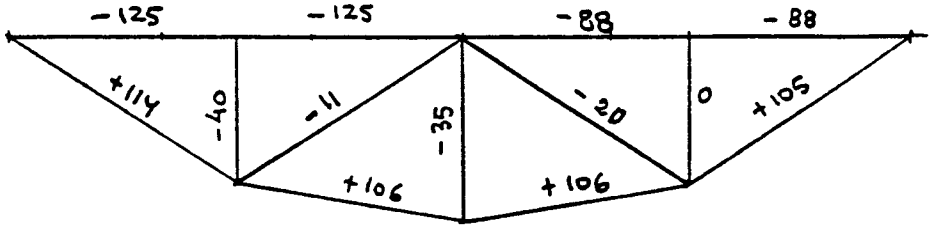


$\rightarrow$ $H - 60 \cdot \cos 60 = 0$ $\qquad H = 30$ kN

$\circlearrowleft B$ $\quad R_A \cdot 12 - 40 \cdot 9 - 60 \sin 60 \cdot 6 - 30 \cdot 3 = 0$ $\qquad R_A = 63{,}5$ kN

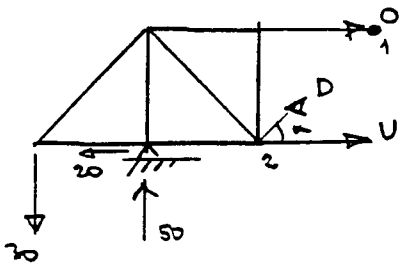
$\uparrow$ $\quad 63{,}5 - 40 - 60 \sin 60 - 30 + R_B = 0$ $\qquad R_B = 58{,}5$ kN





Ex 4:11

$\circlearrowleft A$ $\quad 50\cdot 6 + 20\cdot 3 - R_B\cdot 9 - 30\cdot 3 = 0 \qquad R_B = 30\ \text{kN} \qquad R_A = 50\ \text{kN}$



$\alpha = 45°$

$\uparrow \quad 50 - 30 + D\cdot\cos 45 = 0 \qquad D = -28{,}3$

$\circlearrowleft 1 \quad 50\cdot 6 \ (+20\cdot 3) - 30\cdot 9 - U\cdot 3 = 0 \qquad U = +30\ \text{kN}$

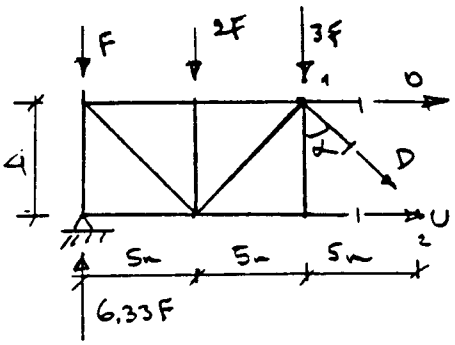
$\circlearrowleft 2 \quad 50\cdot 3 - 30\cdot 6 - O\cdot 3 = 0 \qquad O = +10\ \text{kN}$

Ex 62

Ex 4:12

$\circlearrowleft A \quad 2F\cdot \cancel{5}^{2} + 3F\cdot \cancel{10}^{3} + 3F\cdot \cancel{15}^{5} + F\cdot \cancel{25}^{6} - R_B\cdot \cancel{30} = 0 \qquad R_B = 3{,}67\,F$

$R_A = 6{,}33\,F$



$\uparrow \quad 6{,}33F - 6F - D\cdot\cos\alpha = 0 \qquad \tan\alpha = \frac{5}{4} \qquad \alpha =$

$\circlearrowleft 1 \quad U\cdot 4 + 2F\cdot 5 - 5{,}33\,F\cdot 10 = 0 \qquad D = 0{,}53F$

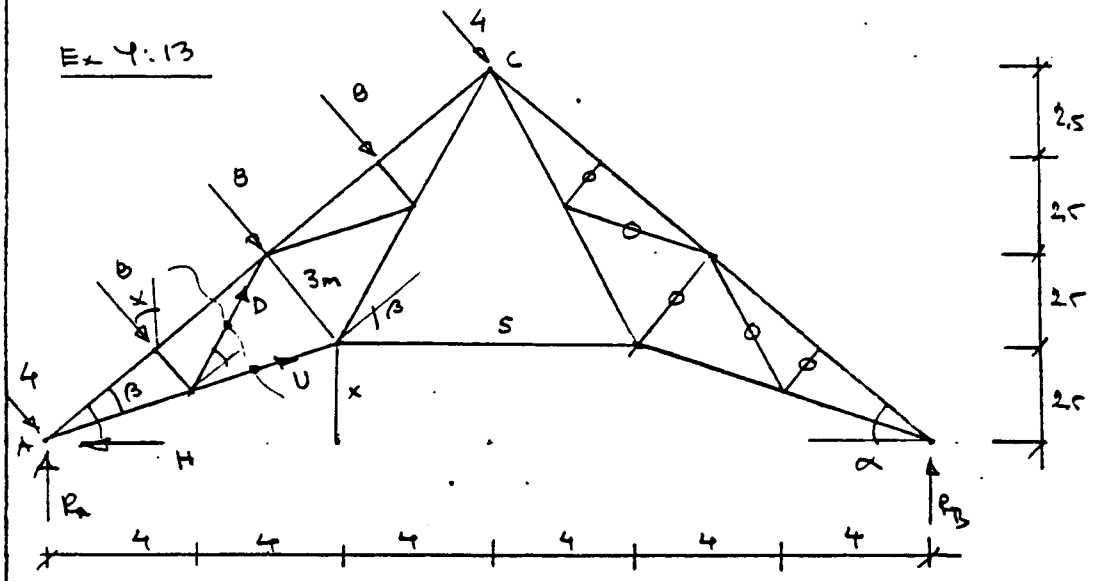
$U = 10{,}8F$

$\circlearrowleft 2 \quad O\cdot 4 + 5{,}33F\cdot 15 - 2F\cdot 10 - 3F\cdot 5 = 0 \qquad O = -11{,}25F$

$V = -3F$

Ex 63

Ex 4:13



$\tan\alpha = \frac{10}{12}$

$\alpha = 39{,}8°$

$AC = 15.62$

$\circlearrowleft A \quad (8 + 2\cdot 8 + 3\cdot 8 + 4\cdot 4)\,\frac{15{,}62}{4} - R_B\cdot 24 = 0 \qquad R_B = 10{,}41\ \text{kN}$

$\uparrow \quad R_A - 10.41 - 32\cdot\cos\alpha = 0 \qquad R_A = 14.17\ \text{kN} \quad \longleftarrow \quad H = 32\cdot\sin\alpha = 20{,}49\ \text{kN}$

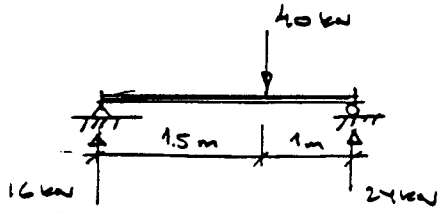
$\tan\beta = \frac{3.2}{15.62} \qquad \beta = 21.0 \qquad \tan(39.8 - 21{,}0) = \frac{x}{8} \qquad x = 2{,}57\ \text{m}$

$\circlearrowleft C \quad S\cdot(10 - 2.57) = 10.41\cdot 12 \qquad S = 16.8\ \text{kN} \qquad V = 0$

$\circlearrowleft \quad U\cos\beta\cdot 3 + \cancel{(8 + 2\cdot 4)}^{4}\,\frac{15.62}{\cancel{4}} - 20.49\cdot 5 - 14.17\cdot 6 = 0 \qquad U = 44.6\ \text{kN}$

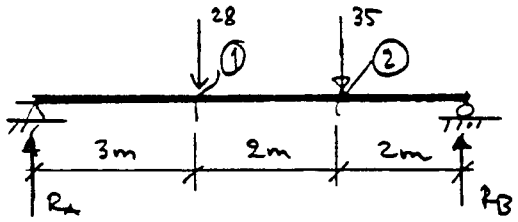
$\circlearrowleft A \quad 8\cdot\frac{15.62}{4} - \underset{1.400}{D\cos\beta}\cdot 1.5 - \underset{1.40}{D\sin\beta}\cdot\frac{15.62}{4} = 0 \qquad D = 11{,}2\ \text{kN}$

Ex 64



$M_{max} = 24 \cdot 1 = \underline{24 \text{ kNm}}$

Ex 65



$\circlearrowleft B \quad R_A \cdot 7 - 28 \cdot 4 - 35 \cdot 2 = 0$

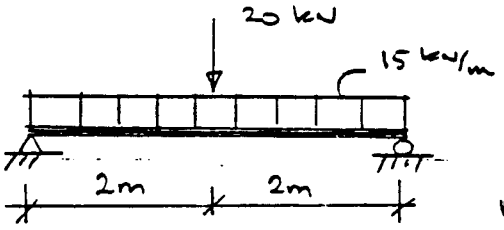
$R_A = 26 \text{ kN}$

$\uparrow \quad R_B = 28 + 35 - 26 = 37 \text{ kN}$

$M_1 = 26 \cdot 3 = \underline{78 \text{ kNm}}$

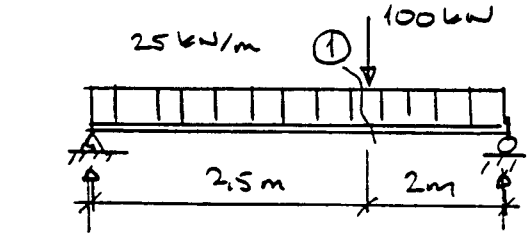
$M_2 = 37 \cdot 2 = 74 \text{ kNm}$

Ex 66



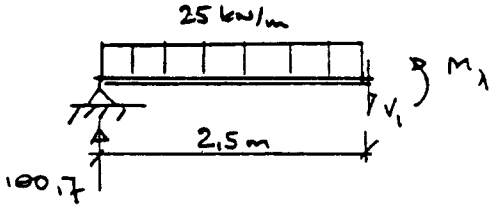
$M_{max} = \frac{20 \cdot 4}{4} + \frac{15 \cdot 4^2}{8} = \underline{50 \text{ kNm}}$

Ex 67



$\circlearrowleft B \quad R_A \cdot 4.5 - 25 \cdot 4.5 \cdot 2.25 - 100 \cdot 2 = 0$

$R_A = 100,7 \text{ kN}$



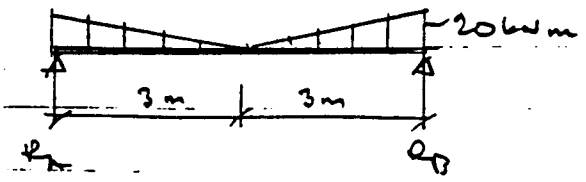
$\uparrow \quad 100,7 - 25 \cdot 2,5 - V_1 = 0$

$V_1 = 38,2 \text{ kN}$

V byter tecken

$M_1 = 100,7 \cdot 2,5 - 25 \cdot 2,5 \cdot 1.25 = \underline{174 \text{ kNm}}$

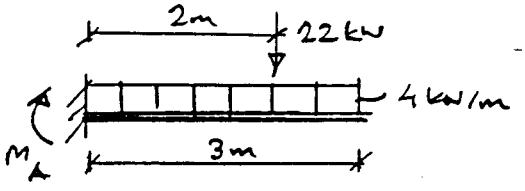
under punktlasten



Ex 68

$$R_A = R_B = \frac{20 \cdot 3}{2} = 30 \text{ kN}$$

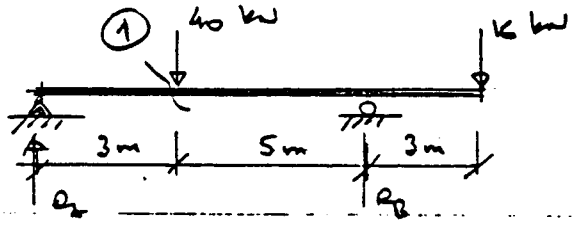
$$M_{max} = 30 \cdot 3 - \frac{20 \cdot 3}{2} \cdot 2 = 30 \text{ kNm}$$



Ex 69

$$\circlearrowleft_A \quad M_A + 22 \cdot 2 + 4 \cdot 3 \cdot 1{,}5 = 0$$

$$M_A = -44 - 18 = -62 \text{ kNm}$$



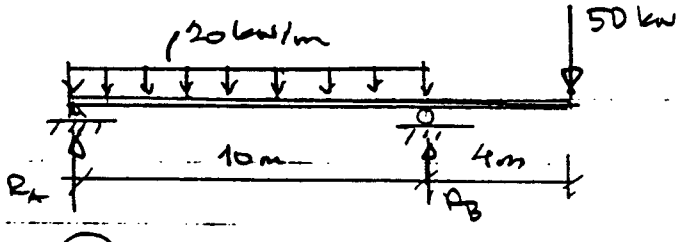
Ex 70

$$\circlearrowleft_B \quad R_A \cdot 8 + 16 \cdot 3 - 40 \cdot 5 = 0$$

$$R_A = 19 \text{ kNm}$$

$$M_A = 19 \cdot 3 = 57 \text{ kNm}$$

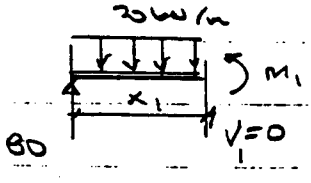
$$M_B = -16 \cdot 3 = -48 \text{ kNm}$$



Ex 71

$$\circlearrowleft_A \quad 20 \cdot 10 \cdot 5 + 50 \cdot 14 - R_B \cdot 10 = 0 \qquad \Rightarrow \quad R_B = 170 \text{ kN}$$

$$\uparrow \quad R_A + 170 - 20 \cdot 10 - 50 = 0 \qquad \Rightarrow \quad R_A = 80 \text{ kN}$$



$$\uparrow \quad 80 - 20 \cdot x_1 = 0$$

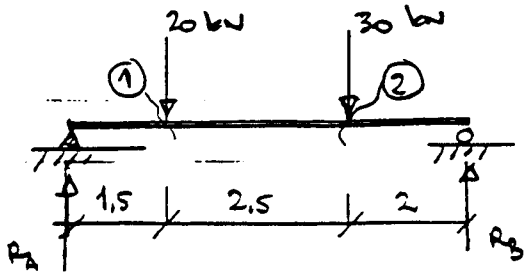
$$x_1 = 4 \text{ m}$$

$$\circlearrowleft \quad M_1 + 20 \cdot 4 \cdot 2 - 80 \cdot 4 = 0$$

$$M_1 = 160 \text{ kNm}$$

$$M_B = -50 \cdot 4 = -200 \text{ kNm}$$

Ex 72



$\circlearrowleft B$ $R_A \cdot 6 - 20 \cdot 4{,}5 - 30 \cdot 2 = 0$

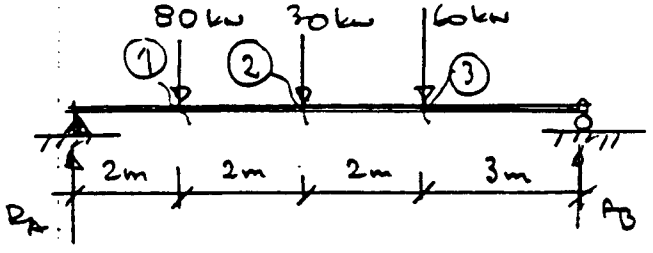
$R_A = 25$ kN

$R_B = 25$ kN

$M_1 = 25 \cdot 1{,}5 = \underline{37{,}5 \text{ kNm}}$

$M_2 = 25 \cdot 2 = \underline{50 \text{ kNm}}$

Ex 73



$\circlearrowleft B$ $R_A \cdot 9 - 80 \cdot 7 - 30 \cdot 5 - 60 \cdot 3 = 0$

$R_A = 99$ kN

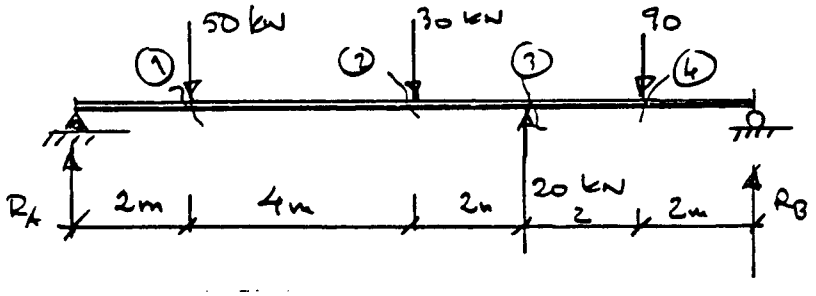
$R_B = 71$ kN

$M_1 = 99 \cdot 2 = \underline{198 \text{ kNm}}$

$M_2 = 99 \cdot 4 - 80 \cdot 2 = \underline{236 \text{ kNm}}$

$M_3 = 71 \cdot 3 = \underline{213 \text{ kNm}}$

Ex 74



$R_A \cdot 12 - 50 \cdot 10 - 30 \cdot 6 + 20 \cdot 4 - 90 \cdot 2 = 0$ $\underline{R_A = 65 \text{ kN}}$

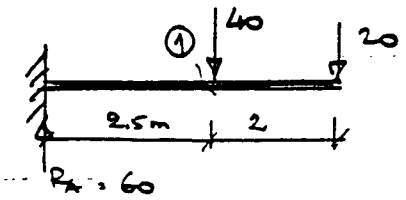
$R_B = 50 + 30 + 90 - 20 - 65 = \underline{85 \text{ kN}}$

$M_1 = 65 \cdot 2 = \underline{130 \text{ kNm}}$

$M_2 = 65 \cdot 6 - 50 \cdot 4 = \underline{190 \text{ kNm}}$

$M_3 = 85 \cdot 4 - 90 \cdot 2 = \underline{160 \text{ kNm}}$

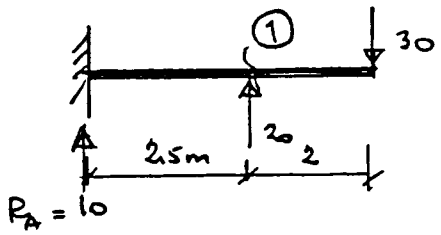
$M_4 = 85 \cdot 2 = \underline{170 \text{ kNm}}$



Ex 75

$M_1 = -20 \cdot 2 = \underline{-40 \text{ kNm}}$

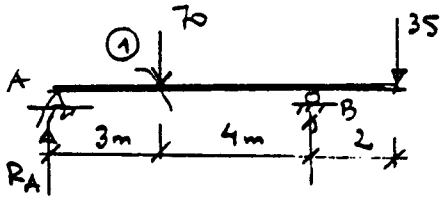
$M_A = -20 \cdot 4.5 - 40 \cdot 2.5 = \underline{-190 \text{ kNm}}$



Ex 76

$M_1 = -30 \cdot 2 = \underline{-60 \text{ kNm}}$

$M_A = -30 \cdot 4.5 + 20 \cdot 2.5 = \underline{-85 \text{ kNm}}$



Ex 77

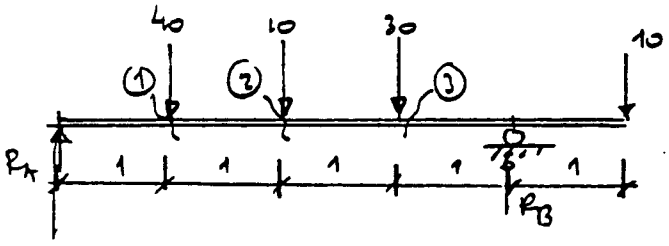
$R_A \cdot 7 - 70 \cdot 4 + 35 \cdot 2 = 0$

$R_A = 30 \text{ kN}$

$R_B = 75 \text{ kN}$

$M_1 = 30 \cdot 3 = \underline{90 \text{ kNm}}$

$M_B = 35 \cdot 2 = \underline{70 \text{ kNm}}$



Ex 78

$\circlearrowleft B \quad R_A \cdot 4 - 40 \cdot 3 - 10 \cdot 2 - 30 \cdot 1 + 10 \cdot 1 = 0$

$\underline{R_A = 40 \text{ kN}}$

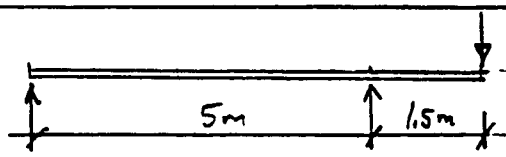
$\underline{R_B = 50 \text{ kN}}$

$M_1 = 40 \cdot 1 = \underline{40 \text{ kNm}}$

$M_2 = 40 \cdot 2 - 40 \cdot 1 = \underline{40 \text{ kNm}}$

$M_3 = -10 \cdot 2 + 50 \cdot 1 = \underline{30 \text{ kNm}}$

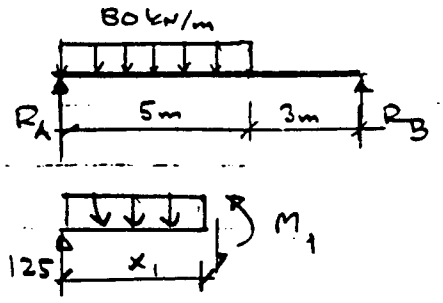
$M_B = \underline{-10 \text{ kNm}}$



Ex 79

$M_B = 20 \cdot 1.5 = 30 \text{ kNm}$

Ex 80



$\circlearrowleft B$ $\quad 80 \cdot 5 \cdot 5{,}5 - R_A \cdot 8 = 0$

$R_A = \underline{275 \text{ kN}}$

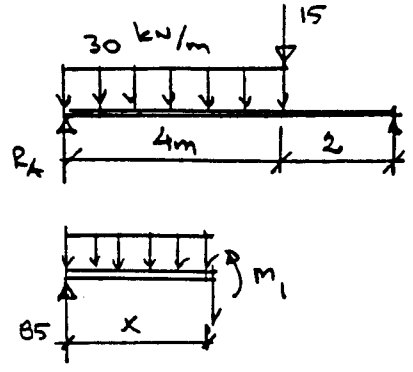
$R_B = \underline{125 \text{ kN}}$

$x = \frac{275}{80} = 3{,}44 \text{ m}$

$M_1 = \frac{275^2}{2 \cdot 80} = \underline{473 \text{ kNm}}$

$M^o = \frac{80 \cdot 25}{8} = \underline{250 \text{ kNm}}$

Ex 81



$\circlearrowleft B$ $\quad R_A \cdot 6 - 30 \cdot 4 \cdot 4 - 15 \cdot 2 = 0$

$R_A = \underline{85 \text{ kN}}$

$R_B = 30 \cdot 4 + 15 - 85 = \underline{50 \text{ kN}}$

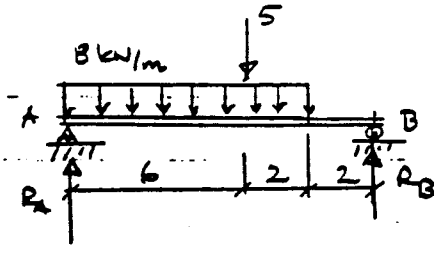
$x = \frac{85}{30} = 2{,}83 \text{ m}$

$M_1^{max} = \frac{85^2}{2 \cdot 30} = \underline{120 \text{ kNm}}$

$M_2 = 50 \cdot 2 = \underline{100 \text{ kN}}$

$M^o = \frac{30 \cdot 4^2}{8} = \underline{60 \text{ kNm}}$

Ex 82



$\circlearrowleft B$ $\quad R_A \cdot 10 - 8 \cdot 8 \cdot 6 - 5 \cdot 4 = 0$

$R_A = \underline{40{,}4 \text{ kN}}$

$R_B = \underline{28{,}6 \text{ kN}}$

$x = \frac{40{,}4}{8} = 5{,}05 \text{ m}$

$M_2 = 28{,}6 \cdot 4 - 8 \cdot 2 \cdot 1 = \underline{98{,}4}$

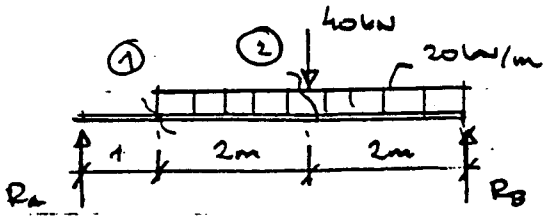
$M_3 = 2 \cdot 28{,}6 = \underline{57{,}2}$

$M_1^{max} = \frac{40{,}4^2}{2 \cdot 8} = \underline{102 \text{ kNm}}$

$M^o_{12} = \frac{8 \cdot 6^2}{8} = \underline{36}$

$M^o_{23} = \frac{8 \cdot 2^2}{8} = \underline{4}$

Ex 83



$R_A \cdot 5 - 40 \cdot 2 - 20 \cdot 4 \cdot 2 = 0$

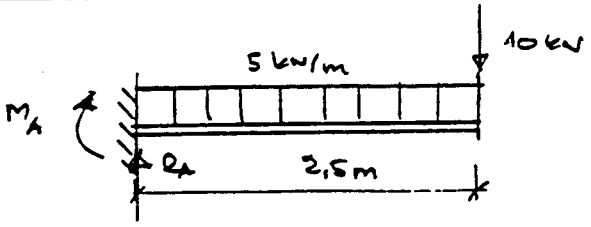
$R_A = 48\ kN \qquad R_B = 72\ kN$

$M_1 = 48 \cdot 1 = 48\ kNm$

$M_2 = 48 \cdot 3 - 20 \cdot 2 \cdot 1 = 104\ kNm$

$M^0 = \frac{20 \cdot 2^2}{8} = 10\ kNm$

Ex 84



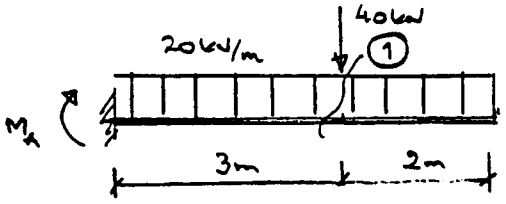
$M_A = -10 \cdot 2{,}5 - \frac{5 \cdot 2{,}5^2}{2} = -40{,}6\ kNm$

$R_A = 10 + 5 \cdot 2{,}5 = 22{,}5\ kN$

$M_1^0 = \frac{5 \cdot 2{,}5^2}{8} = 3{,}9\ kNm$

$M_1 = -10 \cdot 1{,}25 - \frac{5 \cdot 1{,}25^2}{2} = -16{,}4$

Ex 85



$M_1 = -20 \cdot 2 \cdot 1 = -40\ kNm$

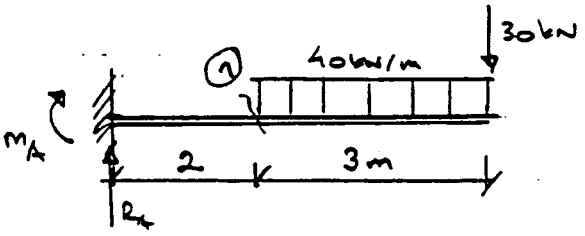
$M_A = -20 \cdot 5 \cdot 2{,}5 - 40 \cdot 3 = -370\ kNm$

$R_A = 20 \cdot 5 + 40 = 140\ kN$

$M^0_{A1} = \frac{20 \cdot 3^2}{8} = 22{,}5$

$M^0_{1B} = \frac{20 \cdot 2^2}{8} = 10$

Ex 86



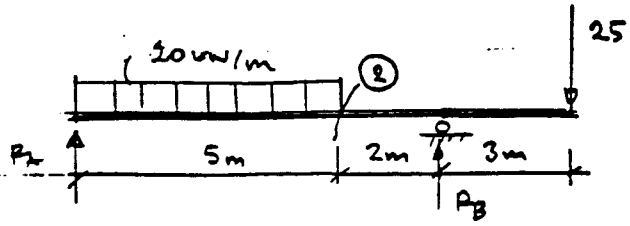
$M_A = -30 \cdot 5 - 40 \cdot 3 \cdot 3{,}5 = -570\ kNm$

$M_1 = -30 \cdot 3 - 40 \cdot 3 \cdot 1{,}5 = -270\ kNm$

$R_A = 40 \cdot 3 + 30 = 150\ kN$

$M^0_{1B} = \frac{40 \cdot 3^2}{8} = 45\ kNm$

Ex 87



$\circlearrowright B$ $R_A \cdot 7 + 25 \cdot 3 - 20 \cdot 5 \cdot 4.5 = 0$ $\qquad R_A = 53{,}6$ kN

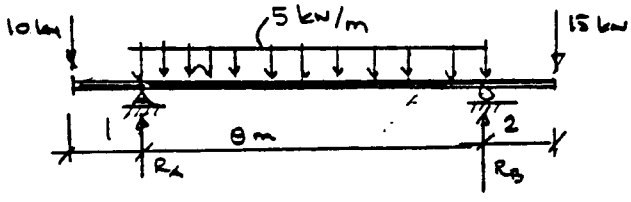
$R_B = 100 + 25 - 53{,}6 = 71.4$ kN

$X_1 = \frac{53{,}6}{20} = 2{,}68$ $\qquad M_{max} = 71.7$ kNm

$M_2 = 71.4 \cdot 2 - 25 \cdot 5 = 17{,}8$ kNm $\qquad M_B = -75$ kNm

$M^0 = \frac{20 \cdot 5^2}{8} = 62{,}5$

Ex 88



$\circlearrowright B$ $R_A \cdot 8 - 5 \cdot 8 \cdot 4 - 10 \cdot 9 + 15 \cdot 2 = 0$

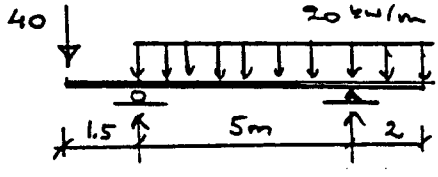
$R_A = 27{,}5$ $\qquad R_B = 40 + 25 - 27{,}5 = 37{,}5$ kN

$M_A = -10 \qquad M_B = -30$

$x = \frac{27{,}5 - 10}{5} = 3{,}5$ m $\qquad M_{max} = 27{,}5 \cdot 3{,}5 - 10 \cdot 4{,}5 - \frac{5 \cdot 3{,}5^2}{2} = 20{,}6$

$M_0 = \frac{5 \cdot 8^2}{8} = 40$ kNm

Ex 89



$M_A = -60 \qquad M_B = -40$

$\circlearrowleft A$ $R_B \cdot 5 - 20 \cdot 7 \cdot 3{,}5 + 40 \cdot 1.5 = 0$

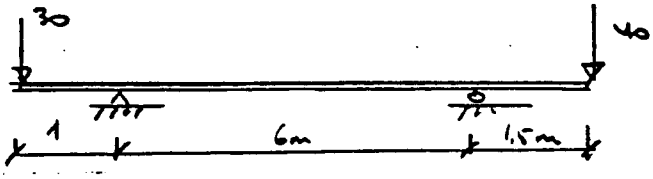
$R_B = 86$ kN $\qquad R_A = 94$ kN

$x = \frac{94 - 40}{20} = 2{,}7$ m

$M_{max} = 94 \cdot 2{,}7 - 40 \cdot 4.2 - \frac{20 \cdot 2{,}7^2}{2} = 12{,}9$ kNm

$M^0_{AB} = \frac{20 \cdot 5^2}{8} = 62{,}5$ kNm $\qquad M^0_{BC} = \frac{20 \cdot 2^2}{8} = 10$ kNm

Ex 90

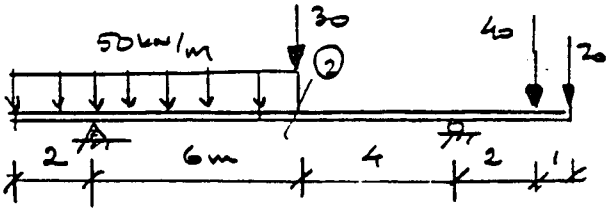


$M_A = -30$ kNm $\qquad M_B = -40 \cdot 1.5 = -60$ kNm

$R_A \cdot 6 - 30 \cdot 7 + 40 \cdot 1.5 = 0 \qquad R_A = 25$ kN

$R_B = 45$ kN

Ex 91



$\circlearrowleft B \quad R_A \cdot 10 - 50 \cdot 8 \cdot 8 - 30 \cdot 4 + 40 \cdot 2 + 20 \cdot 3 = 0$

$R_A = 318$ kN $\qquad R_B = 400 + 90 - 318 = 172$ kN

$M_A = -50 \cdot 2 \cdot 1 = -100 \qquad M_2 = 318 \cdot 6 - 50 \cdot 8 \cdot 4 = 308$ kNm

$M_B = -20 \cdot 3 - 40 \cdot 2 = -140$ kNm

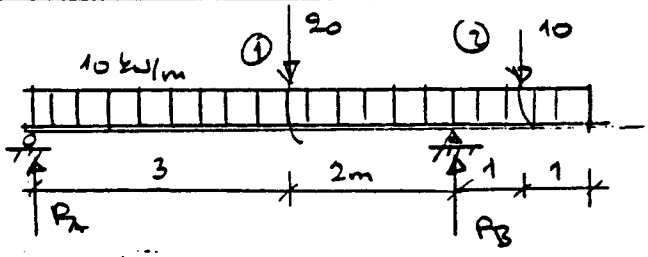
$\uparrow \; 318 - 50(2 + x) = 0 \qquad$ qui $x = 4.36$ m.

$M_{max} = 318 \cdot 4.36 - 50 \cdot 6{,}36 \cdot 3{,}18 = 375$ kNm

$M^0_{0A} = \dfrac{50 \cdot 2^2}{8} = 25 \qquad M^0_{A2} = \dfrac{50 \cdot 6^2}{8} = 225$ kNm

---

Ex 92



$M_2 = -10 \cdot 1 \cdot 0{,}5 = -5$ kNm

$M_B = -10 \cdot 2 \cdot 1 - 10 \cdot 1 = -30$ kN

$\circlearrowleft B \quad R_A \cdot 5 - 20 \cdot 2 + 10 \cdot 1 - 10 \cdot 7 \cdot 1{,}5 = 0$

$R_A = 27$ kN $\qquad R_B = 70 + 30 - 27 = 73$ kN

$\uparrow \; 27 - 10 \cdot x = 0 \qquad x = 2{,}70$

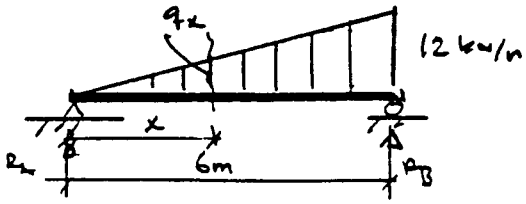
$M_{max} = 27 \cdot 2{,}7 - \dfrac{10 \cdot 2{,}7^2}{2} = 36{,}5$ kNm

$M_1 = 27 \cdot 3 - 10 \cdot 3 \cdot 1.5 = 36$ kNm

$M^0_{A1} = \dfrac{10 \cdot 3^2}{8} = 11{,}25 \qquad M_{1B} = \dfrac{10 \cdot 2^2}{8} = 5$ kNm

$M^0 = \dfrac{10 \cdot 1^2}{8} = 1{,}25$

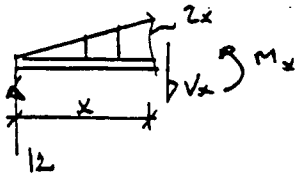
Ex 93



$$\circlearrowright B \quad R_A \cdot 6 - \frac{12 \cdot 6}{2} \cdot 2 = 0$$

$$R_A = 12 \text{ kN} \qquad R_B = 24 \text{ kN}$$

$$\frac{q_x}{x} = \frac{12}{6} \qquad q_x = 2x$$



$$\uparrow \quad 12 - \frac{2x \cdot x}{2} - V_x = 0$$

$$\circlearrowleft \quad M_x - 12 \cdot x + \frac{2x \cdot x}{2} \cdot \frac{x}{3} = 0$$

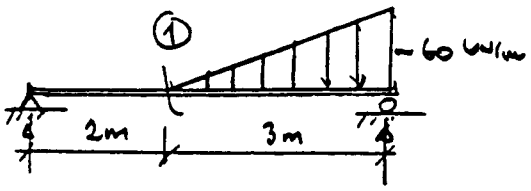
$$V_x = 12 - x^2$$

$$M_x = 12x - \frac{x^3}{3}$$

$$M_{max} = 12 \cdot \sqrt{12} - \frac{12\sqrt{12}}{3} = 27{,}7$$

x	V_x	M_x
0	12	0
1	11	11,7
2	8	21,3
3	3	27
4	-4	26,7
5	-13	18,3
6	-24	0

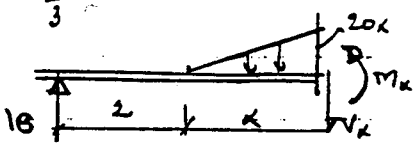
Ex 94



$$R_A \cdot 5 - \frac{60 \cdot 3}{2} \cdot 1 = 0 \qquad R_A = 18 \text{ kN}$$

$$R_B = 72 \text{ kN}$$

$$\frac{q_x}{x} = \frac{60}{3}$$



$$\uparrow \quad 18 - \frac{20x \cdot x}{2} - V_x = 0$$

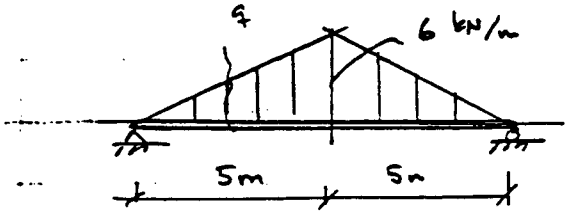
$$\circlearrowleft \quad M_x + \frac{20x \cdot x}{2} \cdot \frac{x}{3} - 18(2+x) = 0$$

$$V_x = 18 - 10x^2 \qquad V_x = 0 \text{ per } x = \sqrt{1{,}8} = 1{,}34 \text{ m}$$

$$M_x = 36 + 18x - \frac{10x^3}{3} \qquad M_{max} = 52{,}1 \text{ kNm}$$

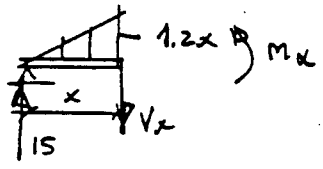
x	V_x	M_x
0	18	36
1	8	50,7
2	-22	45,3
3	-72	0

Ex 95



$$\frac{q_x}{x} = \frac{6}{5} \qquad q = 1{,}2x$$

$$R_A = R_B = \frac{6 \cdot 10}{2 \cdot 2} = 15 \text{ kN}$$



$$\uparrow \; 15 - \frac{1{,}2x \cdot x}{2} - V_x = 0$$

$$\circlearrowleft \; M_x + \frac{1{,}2x \cdot x}{2} \cdot \frac{x}{3} - 15x = 0$$

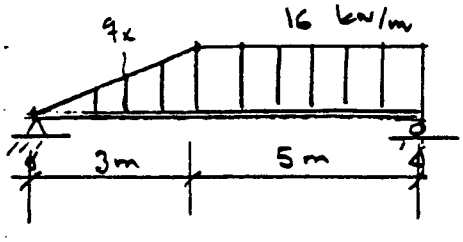
$$V_x = 15 - 0{,}6 \cdot x^2$$

$$M_x = 15x - 0{,}2x^3$$

$$\underline{M_{max} = 50 \text{ kNm}}$$

x	V_x	M_x
0	15	0
1	14,4	14,8
2	12,6	28,4
3	9,6	39,6
4	5,4	47,2
5	0	50

Ex 96



$R_B = 64$

$$\frac{q_x}{x} = \frac{16}{3} \qquad q_x = \frac{16}{3}x$$

$$\uparrow \; 43 - \frac{16x}{3} \cdot \frac{x}{2} - V_x = 0$$

$$\circlearrowleft \; M_x - 43x + \frac{16x}{3} \cdot \frac{x}{2} \cdot \frac{x}{3} = 0$$

$$V_x = 43 - \frac{8x^2}{3}$$

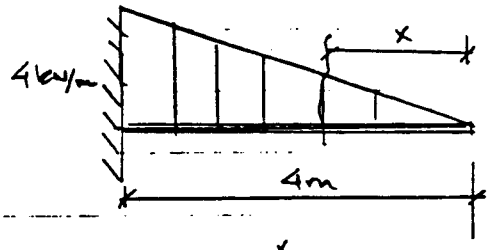
$$M_x = 43x - \frac{8x^3}{9}$$

x	V_x	M_x
0	43	0
1	40,3	42,1
2	32,3	78,9
3	19	105

$$M_{max} = \frac{64^2}{2 \cdot 16} = \underline{116{,}3 \text{ kNm}}$$

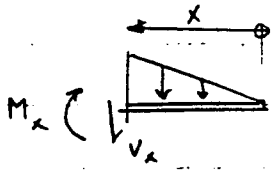
$$M_0 = \frac{16 \cdot 5^2}{8} = \underline{50 \text{ kNm}}$$

Ex97



$$M_A = \frac{4 \cdot 4}{2} \cdot \frac{4}{3} = 10{,}7 \text{ kNm}$$

$$\frac{q_x}{x} = \frac{4}{4} \qquad q_x = 1 \cdot x$$



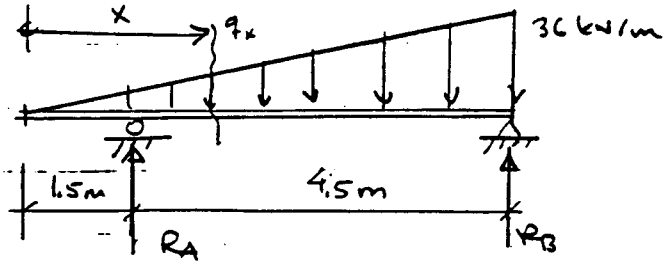
$$\downarrow V_x + \frac{1 \cdot x \cdot x}{2} = 0$$

$$\circlearrowright M_x + \frac{x \cdot x}{2} \cdot \frac{x}{3} = 0$$

$$V_x = -\frac{x^2}{2} \qquad M_x = -\frac{x^3}{6}$$

x	V_x	M_x
0	0	0
1	+0,5	-0,2
2	+2	-1,3
3	+4,5	-4,5
4	+8	-10,7

Ex98



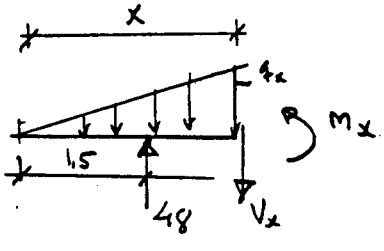
$$\frac{q_x}{x} = \frac{36}{6}$$

$$q_x = 6x$$

$$\circlearrowright B \quad R_A \cdot 4{,}5 - \frac{36 \cdot 6}{2} \cdot \frac{6}{3} = 0$$

$$R_A = 48 \text{ kN}$$

$$R_B = 60 \text{ kN}$$



$$\downarrow V_x - 48 + \frac{6x \cdot x}{2} = 0$$

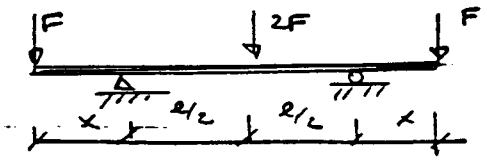
$$\circlearrowleft M_x + \frac{6x \cdot x}{2} \cdot \frac{x}{3} - 48(x - 1{,}5)$$

$$V_x = 48 - 3x^2$$

$$M_x = 48(x - 1{,}5) - x^3$$

x	V_x	M_x
1,5	41,25	-3,4
2	36	+16
3	21	+45
4	0	+56
5	-27	+43
6	-60	0

Ex 99



$R_A = R_B = 2F$

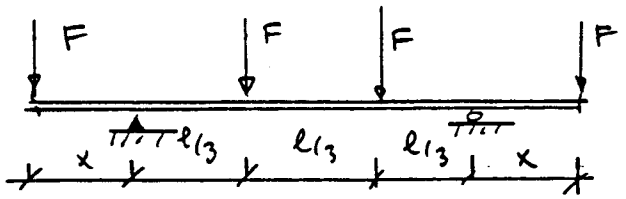
$M_{\text{střed}} = F \cdot x$ (dříve)

$M_{\text{střed}} = 2F \cdot \frac{l}{2} - F\left(x + l/2\right)$

$x = l - x - l/2$

$2x = l/2 \qquad \underline{x = \frac{l}{4}}$

Ex 100



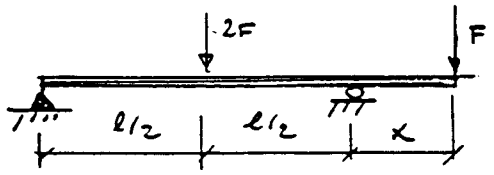
$R_A = R_B = 2F$

$M_{\text{střed}} = Fx \qquad M_{\text{střed}} = 2F \cdot l/3 - F\left(x + l/3\right)$

$x = \frac{2l}{3} - x - \frac{l}{3}$

$2x = \frac{l}{3} \qquad \underline{x = \frac{l}{6}}$

Ex 101



$\circlearrowleft B \quad R_A \cdot l - 2F \cdot l/2 + Fx = 0$

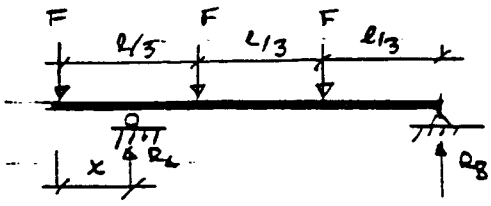
$R_A = F - \frac{Fx}{l}$

$M_{\text{střed}} = F \cdot x \qquad M_{\text{střed}} = \left(F - \frac{Fx}{l}\right)\frac{l}{2} - F\left(\frac{l}{2} - \frac{x}{2}\right)$

$x = \frac{l}{2} - \frac{x}{2}$

$\underline{x = \frac{l}{3}}$

Ex 102



$$\circlearrowleft A \quad F\left(\frac{l}{3}-x\right) + F\left(\frac{2l}{3}-x\right) - Fx - R_B(l-x) = 0$$

$$Fl - F\cdot 2x = Fx + R_B(l-x)$$

$$F(l-3x) = R_B(l-x) \qquad R_B = \frac{(l-3x)}{l-x}F$$

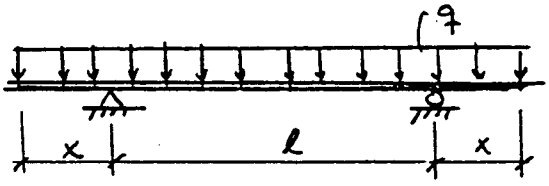
$$M_{střed} = F\cdot x \qquad M_{střed} = R_B\cdot\frac{l}{3} = \frac{(l-3x)}{l-x}\cdot\frac{l}{3}\cdot F$$

$$x = \frac{l^2-3xl}{3(l-x)} \qquad lx - x^2 = \frac{l^2}{3} - xl$$

$$x^2 - 2xl + \frac{l^2}{3} = 0$$

$$x = +l \pm\sqrt{l^2-\frac{l^2}{3}} = l \mp l\sqrt{\frac{2}{3}} = \underline{l\left(1-\sqrt{\frac{2}{3}}\right) = 0{,}1835l}$$

Ex 103



$$R_A = R_B = q\cdot\left(x+\frac{l}{2}\right)$$

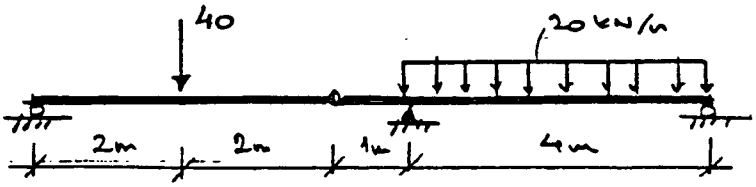
$$M_{střed} = \frac{qx^2}{2} \qquad M_{střed} = q\left(x+\frac{l}{2}\right)\cdot\frac{l}{2} - \frac{q}{2}\left(x+\frac{l}{2}\right)^2 =$$

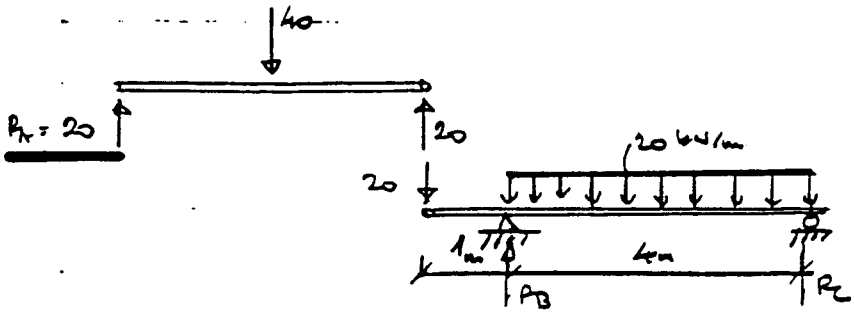
$$= \frac{q}{2}\left(\cancel{xl} + \frac{l^2}{2} - x^2 - \frac{l^2}{4} - \cancel{lx}\right) = \frac{q}{2}\left(\frac{l^2}{4} - x^2\right)$$

$$\frac{\cancel{q}x^2}{\cancel{2}} = \frac{\cancel{q}}{\cancel{2}}\left(\frac{l^2}{4} - x^2\right)$$

$$2x^2 = \frac{l^2}{4} \qquad \underline{x = \frac{l}{\sqrt{8}} = 0{,}354l}$$

Ex 104





$M_{AB} = 20 \cdot 2 = 40$ kNm

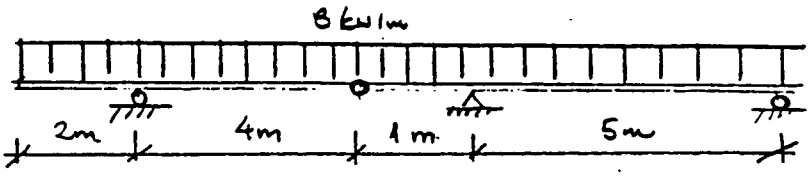
$\circlearrowleft C \quad R_B \cdot 4 - 20 \cdot 5 - 20 \cdot 4 \cdot 2 = 0 \qquad R_B = 65$ kN

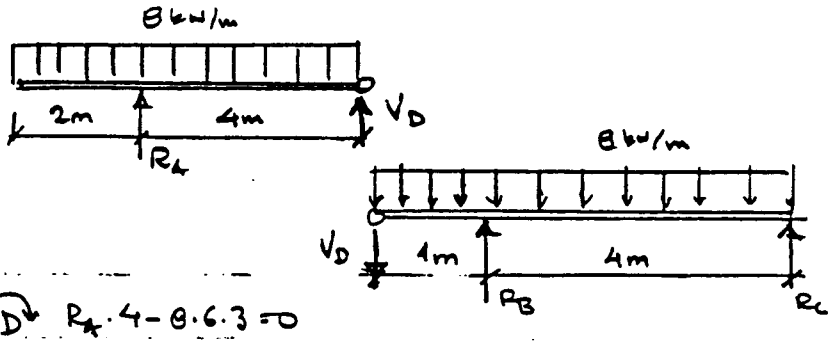
$\uparrow \quad R_A + R_B - 20 - 20 \cdot 4 = 0 \qquad R_C = 35$ kN

$x = \frac{35}{20} = 1{,}75$ m $\qquad M_{max} = 35 \cdot 1{,}75 - \frac{20 \cdot 1{,}75^2}{2} = 30{,}6$ kNm

$M^0_{BC} = \frac{20 \cdot 4^2}{8} = 40$ kNm

Ex 105





$\circlearrowleft D \quad R_A \cdot 4 - 8 \cdot 6 \cdot 3 = 0$

$R_A = 36$ kN $\qquad V_D = 12$ kN

$\circlearrowleft C \quad R_B \cdot 4 - 8 \cdot 5 \cdot 2{,}5 - 12 \cdot 5 = 0$

$R_B = 40$ kN $\qquad R_C = 12$ kN

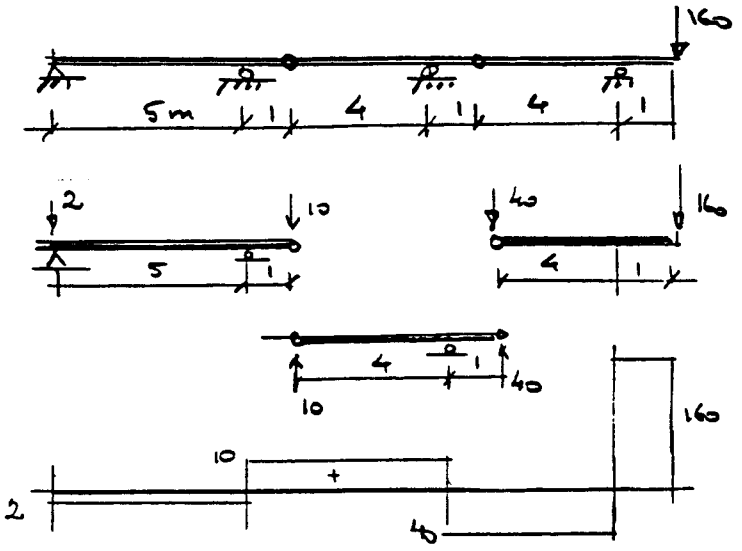
$M_A = -8 \cdot 2 \cdot 1 = -16 \qquad M^0_A = \frac{8 \cdot 2^2}{8} = 4$

$M^{max}_{AD} = \frac{12^2}{16} = 9$ kNm $\qquad M^0_{AB} = \frac{8 \cdot 5^2}{8} = 25$

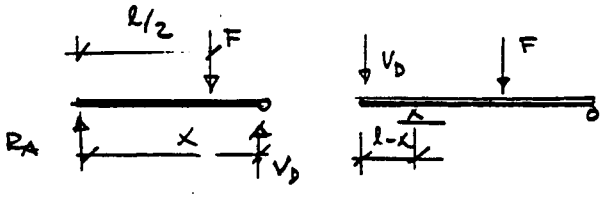
$M_B = -12 \cdot 1 - 8 \cdot 1 \cdot 0{,}5 = -16$ kNm

$M^{max}_{BC} = \frac{12^2}{16} = 9$ kNm $\qquad M^0_{BC} = \frac{8 \cdot 4^2}{8} = 16$ kNm

Ex 106



Ex 107



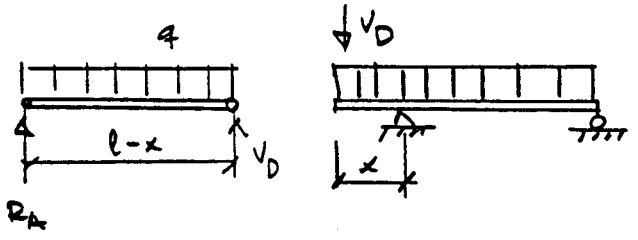
$\curvearrowright A$ $\quad F\cdot\frac{l}{2} - V_D\cdot x = 0 \qquad V_D = \frac{Fl}{2x} \qquad M_B = \frac{Fl}{2x}(l-x)$

$\curvearrowright D$ $\quad R_A\cdot x - F\left(x-\frac{l}{2}\right) = 0 \qquad R_A = \frac{F\left(x-\frac{l}{2}\right)}{x} \qquad M_{feld} = \frac{F\left(x-\frac{l}{2}\right)}{x}\cdot\frac{l}{2}$

Hieraus erhalten $\quad l - x = x - \frac{l}{2}$

$$2x = \frac{3l}{2} \qquad \underline{x = \frac{3l}{4}}$$

Ex 108



$$M_{feld} = \frac{q(l-x)^2}{8} = \frac{q}{8}\left(l^2 - 2lx + x^2\right)$$

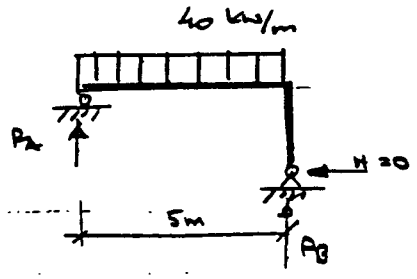
$$V_D = \frac{q(l-x)}{2} \qquad M_{Stütz} = \frac{q(l-x)}{2}\cdot x + \frac{qx^2}{2} = \frac{q}{2}\left(lx - x^2 + x^2\right) = \frac{qlx}{2}$$

$$2\cdot\frac{q}{8}\left(l^2 - 2lx + x^2\right) = \frac{qlx}{2}$$

$$l^2 - 2lx + x^2 = 2lx \qquad x = 2l \pm \sqrt{4l^2 - l^2} = 2l - \sqrt{3}l$$

$$x^2 - 4lx + l^2 = 0 \qquad = \underline{l\left(2-\sqrt{3}\right) = 0{,}268\,l}$$

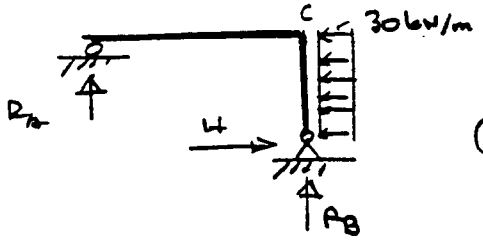
Ex 109



$R_A = R_B = \frac{40 \cdot 5}{2} = 100$ kNm

$M_{max} = \frac{40 \cdot 5^2}{8} = \underline{125 \text{ kNm}}$

Ex 110



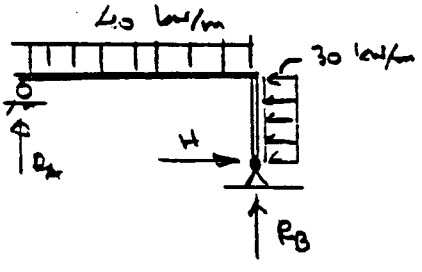
$H = 30 \cdot 2 = 60$ kN

$\circlearrowleft B \quad R_A \cdot 5 - 30 \cdot 2 \cdot 1 = 0$

$R_A = 12 \qquad R_B = -12$

$\underline{M_C = 12 \cdot 5 = 60 \text{ kNm}}$ (dök)

Ex 111



$\circlearrowleft B \quad R_A \cdot 5 - 40 \cdot 5 \cdot 2{,}5 - 30 \cdot 2 \cdot 1 = 0$

$R_A = 112$ kN

$\uparrow -200 + 112 + R_B = 0 \qquad R_B = 88$ kN

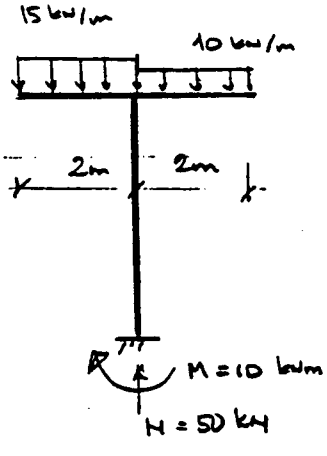
$H = 60$ kN

$M_C = 112 \cdot 5 - 40 \cdot 5 \cdot 2{,}5 = \underline{60 \text{ kNm}}$

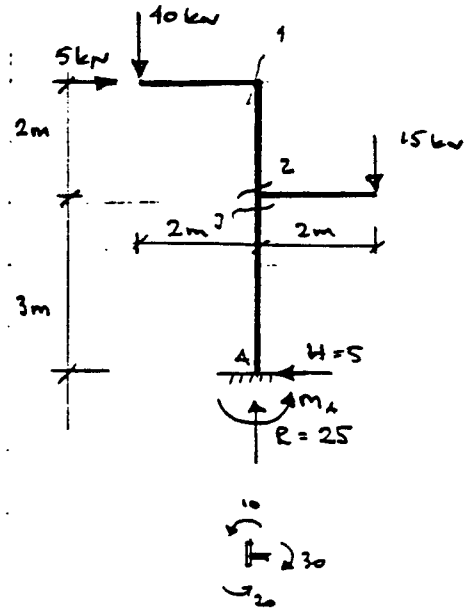
$M^0 = \frac{40 \cdot 5^2}{8} = 125$ kNm

$K_0 = \frac{11{,}2}{4} = 2{,}8$ m

Ex 112



Ex 113



$\circlearrowleft A$ $\quad M_A + 10\cdot 2 - 5\cdot 5 - 15\cdot 3 = 0$

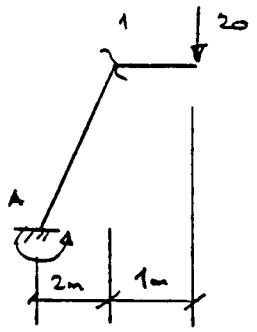
$M_A = 35$ kNm (d.v.s)

$M_1 = 10\cdot 2 = 20$ kNm (dsk)

$M_2 = 10\cdot 2 - 5\cdot 2 = 10$ kNm (d.e.s)

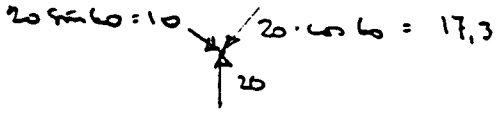
$M_3 = 35 - 5\cdot 3 = 20$ kNm (d.v.s)

Ex 114

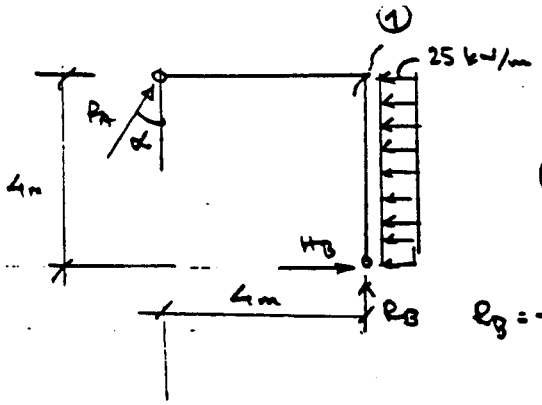


$M_1 = 20$ kNm (dsk)

$M_A = 20\cdot 3 = 60$ kNm (dvs)



Ex 115



$\tan\alpha = \frac{2}{3}$ $\quad \alpha = 33{,}7°$

$\circlearrowright B$ $\quad R_A\cdot\cos\alpha\cdot 4 + R_A\cdot\sin\alpha\cdot 4 - 25\cdot 4\cdot 2 = 0$

$R_A = 36$ kN

$R_B = -R_A\cos 33{,}7 = -30$ kN

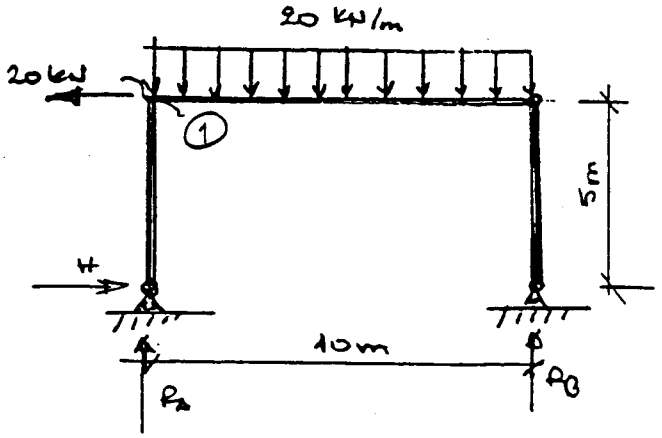
$R_A \sin 33{,}7 = 25$ kN

$H = 25\cdot 4 - 20 = 80$ kN

$M_1 = 80\cdot 4 - 25\cdot 4\cdot 2 = 120$ kNm

$y = \frac{80}{25} = 3{,}2$ $\quad M_{A1}^{max} = 80\cdot 3{,}2 - 25\cdot 3{,}2\cdot 1{,}6 = 128$ kNm

Ex 116



$\circlearrowleft A \quad -20\cdot 5 + 20\cdot 10\cdot 5 - R_B\cdot 10 = 0$

$R_B = 90$ kN

$\uparrow \quad R_A + R_B - 200 = 0$

$R_A = 110$ kN

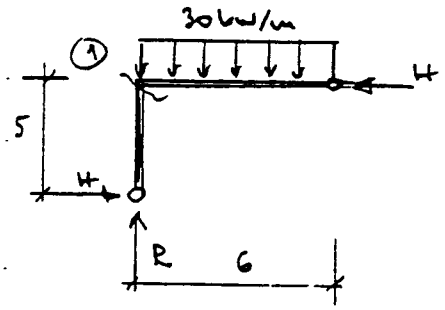
$H = 20$ kN

$M_1 = 20\cdot 5 = \underline{100 \text{ kNm}}$ (dök)

$x = \frac{90}{20} = 4{,}5$ m $\quad M_{max} = 90\cdot 4{,}5 - 20\cdot 4{,}5\cdot 2{,}25 = \underline{202{,}5 \text{ kNm}}$

$M_0 = \frac{20\cdot 100}{8} = \underline{250 \text{ kNm}}$

Ex 117



$\circlearrowleft A \quad 30\cdot 6\cdot 3 - H\cdot 5 = 0$

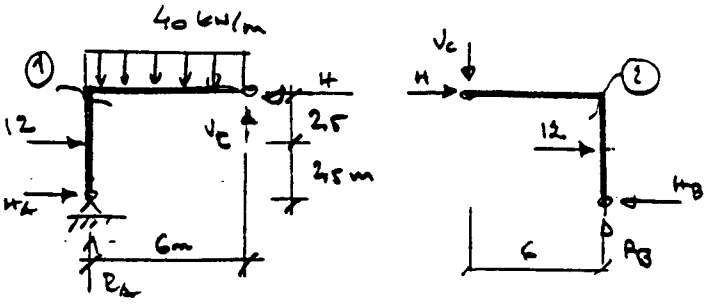
$H = 108$ kN

$R_A = 30\cdot 6 = 180$ kN $= R_B$

$M_1 = 108\cdot 5 = \underline{540 \text{ kNm}}$

$M_0 = \frac{30\cdot 12^2}{8} = 540$

Ex 118



Helaramen $\circlearrowleft B \quad R_A\cdot 12 + 12\cdot 2{,}5\cdot 2 - 40\cdot 6\cdot 9 = 0 \qquad R_A = 175 \quad R_B = 65$

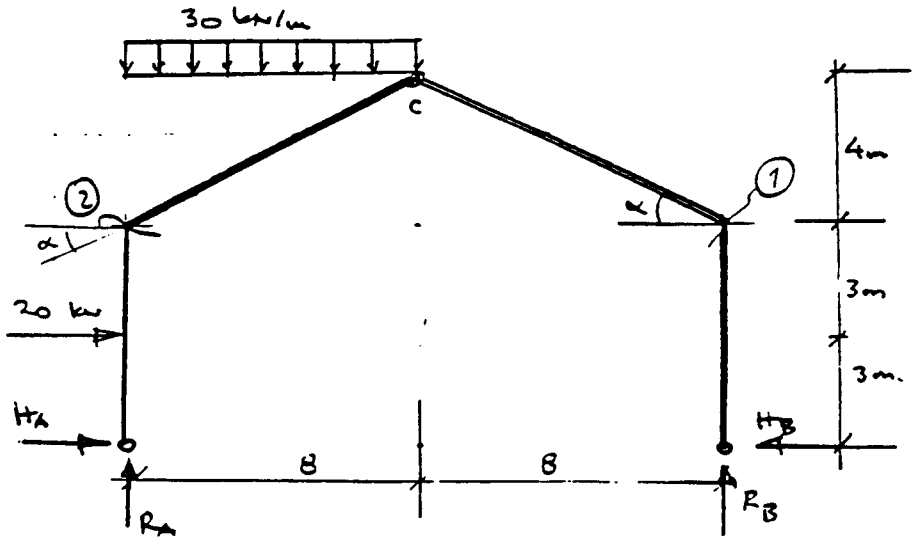
$\circlearrowleft C \quad H_B\cdot 5 - 65\cdot 6 - 12\cdot 2{,}5 = 0 \qquad H_B = 84$ kN $\qquad H_A = 60$ kN

$M_1 = 60\cdot 5 + 12\cdot 2{,}5 = \underline{330 \text{ (dök)}} \qquad M_2 = 84\cdot 5 - 12\cdot 2{,}5 = \underline{390 \text{ dök}}$

$x = \frac{V_C}{40} = \frac{65}{40} = 1{,}625 \qquad M_{1C}^{max} = \frac{65^2}{2\cdot 40} = \underline{53 \text{ kNm}}$

$M_{1C}^{0} = \frac{40\cdot 6^2}{8} = \underline{180 \text{ kNm}}$

Ex 119



$\tan\alpha = \frac{4}{8}$

$\sin\alpha = 0{,}447$

$\cos\alpha = 0{,}894$

(A) $20\cdot 3 + 30\cdot 8\cdot 4 - R_B\cdot 16 = 0$ $\quad R_B = 63{,}8 \quad R_A = 176{,}2$

(C) $H_B\cdot 10 - 63{,}8\cdot 8 = 0$ $\quad H_B = 51{,}0$ kN $\quad H_A = 31$ kN

(2) $M_2 = 31\cdot 6 + 20\cdot 3 = \underline{\underline{246 \text{ kNm}}}$

$N_2 = -176{,}2\cdot 0{,}447 - 51\cdot 0{,}894 = -124$

$V_2 = +176{,}2\cdot 0{,}894 - 51\cdot 0{,}447 = +135$

(1) $M_1 = 51\cdot 6 = \underline{\underline{306 \text{ kNm}}}$

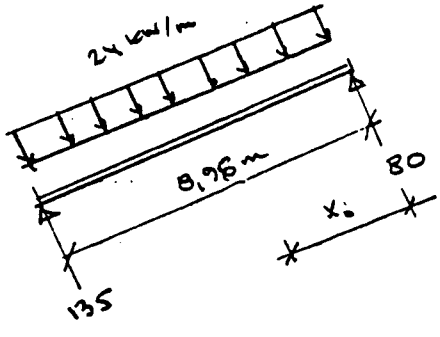
$N_1 = -63{,}8\cdot 0{,}447 - 51\cdot 0{,}894 = -74$

$V_1 = -63{,}8\cdot 0{,}894 + 51\cdot 0{,}447 = -34$

(C) $N_C = -51\cdot 0{,}894 + 63{,}8\cdot 0{,}447 = -17$

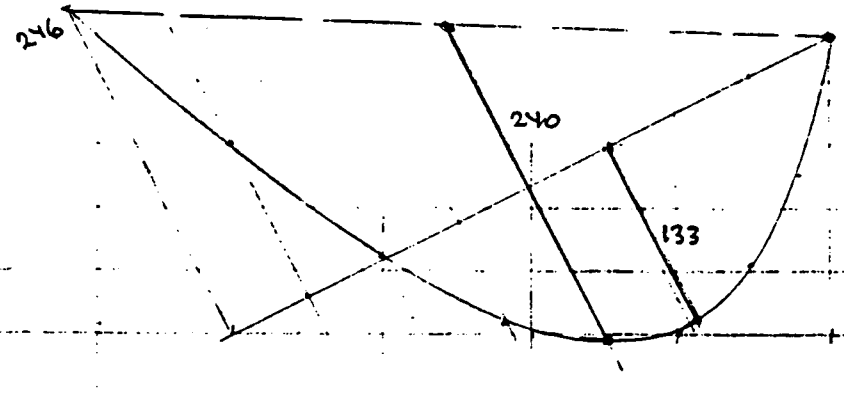
$V_C = -63{,}8\cdot 0{,}894 - 51\cdot 0{,}447 = -79{,}8$ $\quad M^0 = \frac{30\cdot 8^2}{8} = 240$

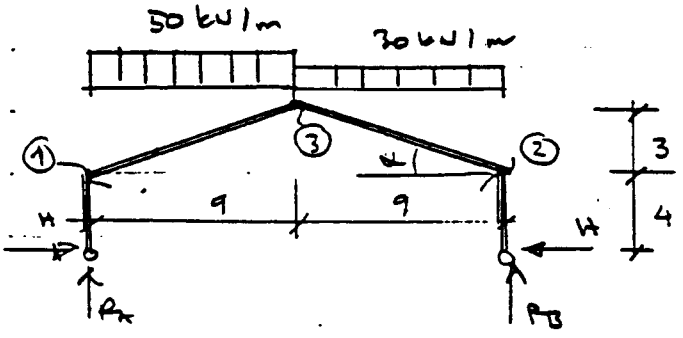
$q = \frac{30\cdot \cancel{8}\cdot \cos^2\alpha}{\cancel{8}} = 24$ kN/m



$x_0 = \frac{80}{24} = 3{,}33$, horisontalt $= 2{,}98$ m

$M_{max} = \frac{80^2}{2\cdot 24} = \underline{\underline{133 \text{ kNm}}}$





$\tan\alpha = \frac{3}{9}$

$\sin\alpha = 0{,}316$

$\cos\alpha = 0{,}949$

B: $R_A \cdot 18 - 50 \cdot 9 \cdot 13{,}5 - 30 \cdot 9 \cdot 4{,}5 = 0$ $\quad R_A = \underline{405 \text{ kN}}$ - $R_B = \underline{315 \text{ kN}}$

C: $30 \cdot 9 \cdot 4{,}5 + H \cdot 7 - 315 \cdot 9 = 0$ $\quad H = \underline{231{,}4 \text{ kN}}$

1 $\quad M_1 = 231{,}4 \cdot 4 = \underline{926 \text{ kNm}}$

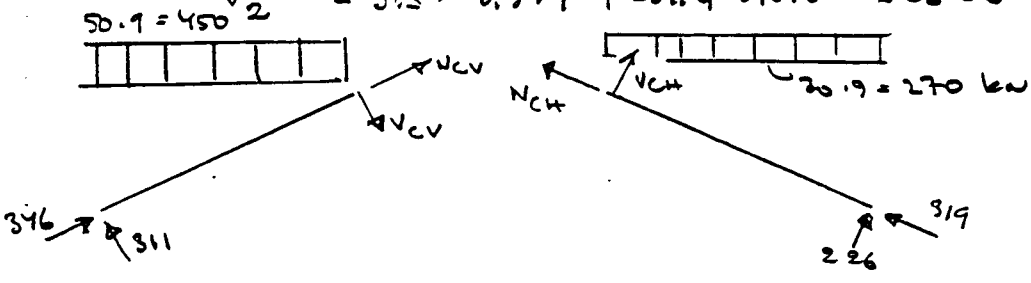
$N_1 = -231{,}4 \cdot 0{,}949 - 405 \cdot 0{,}316 = -348 \text{ kN}$

$V_1 = 405 \cdot 0{,}949 - 231{,}4 \cdot 0{,}316 = +311 \text{ kN}$

2 $\quad M_2 = \underline{926 \text{ kNm}}$

$N_2 = -231{,}4 \cdot 0{,}949 - 315 \cdot 0{,}316 = -319 \text{ kN}$

$V_2 = -315 \cdot 0{,}949 + 231{,}4 \cdot 0{,}316 = -226 \text{ kN}$



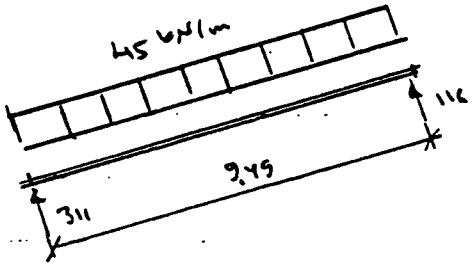
$\nearrow \quad N_{CV} + 346 - 450 \cdot 0{,}316 = 0 \qquad N_{CV} = -204 \text{ kN}$

$\searrow \quad V_{CV} - 311 + 450 \cdot 0{,}949 = 0 \qquad V_{CV} = -116 \text{ kN}$

$\nwarrow \quad N_{CH} + 319 - 270 \cdot 0{,}316 = 0 \qquad N_{CH} = -234 \text{ kN}$

$\uparrow \quad V_{CH} + 226 - 270 \cdot 0{,}949 = 0 \qquad V_{CH} = +30$

$M^0_{13} = \frac{50 \cdot 9^2}{8} = 506 \qquad M^0_{32} = \frac{30 \cdot 9^2}{8} = 304$

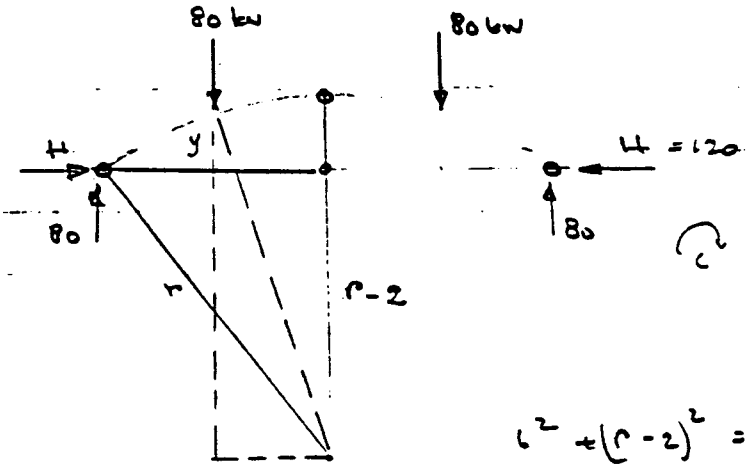


$q = \frac{50 \cdot 9 \cdot \cos^2\alpha}{9} = 45{,}0$

$x_0 = \frac{116}{45} = 2{,}58 \text{ m}$

$M_{max} = \frac{116^2}{2 \cdot 45} = \underline{150 \text{ kNm}}$

Ex 121



$\circlearrowleft C \quad 80 \cdot 3 + H \cdot 2 - 80 \cdot 6 = 0$

$\underline{H = 120 \text{ kN}}$

$6^2 + (r-2)^2 = r^2$

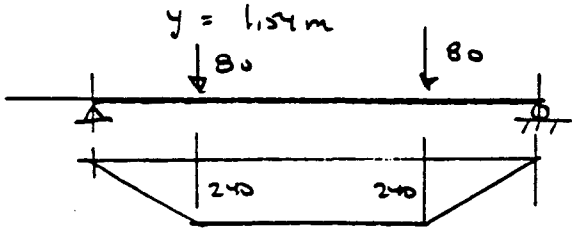
$36 + \cancel{r^2} + 4 - 4r = \cancel{r^2}$

$\underline{r = 10 \text{ m}} \qquad r - 2 = 8 \text{ m}$

$(8+y)^2 + 3^2 = 10^2$

$8 + y = \sqrt{91}$

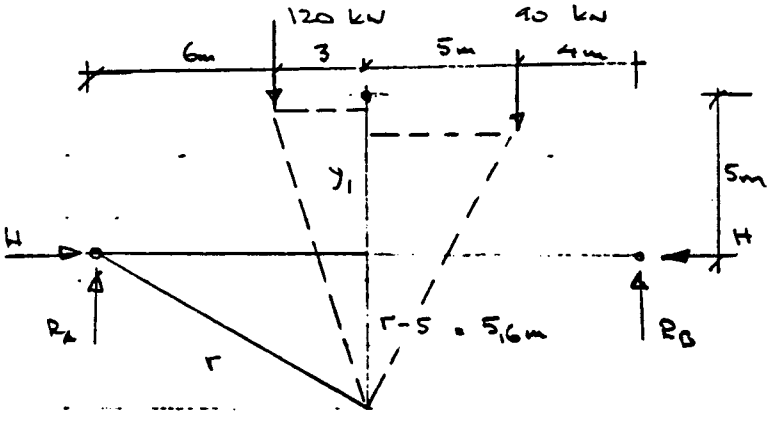
$y = 1{,}54 \text{ m}$



$\eta_1 = \frac{M_1}{H} = \frac{240}{120} = 2{,}0 \text{ m}$

$M_1 = 120(2{,}0 - 1{,}54) = \underline{55 \text{ kNm}}$

Ex 122



$\circlearrowright B \quad R_A \cdot 10 - 120 \cdot 12 - 90 \cdot 4 = 0$

$\underline{R_A = 100 \text{ kN}} \quad \underline{R_B = 110 \text{ kN}}$

$H \cdot 5 + 120 \cdot 3 - 100 \cdot 9 = 0 \quad \underline{H = 108}$

$r^2 = 9^2 + (r-5)^2$

$\cancel{r^2} = 81 + \cancel{r^2} + 25 - 10r$

$\underline{r = 10{,}6 \text{ m}}$

$(5{,}6 + y_1)^2 + 3^2 = 10{,}6^2$

$y_1 = 4{,}5666 \text{ m}$

$\eta_1 = \frac{M_1^0}{H} = \frac{100 \cdot 6}{108} = 5{,}556$

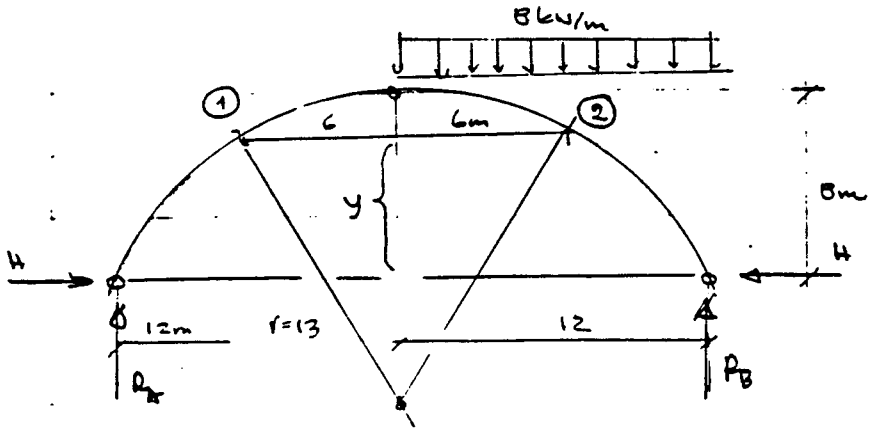
$M_1 = 108(5{,}552 - 4{,}567) = \underline{+107 \text{ kNm}}$

$(5{,}6 + y_2)^2 + 5^2 = 10{,}6^2$

$y_2 = 3{,}747 \text{ m}$

$\eta_2 = \frac{110 \cdot 4}{108} = 4{,}074 \text{ m}$

$M_2 = 108(4{,}074 - 3{,}747) = \underline{35 \text{ kNm}}$



Ex 123

$\circlearrowleft B \quad R_A \cdot 24 - 8 \cdot 12 \cdot 6 = 0$

$R_A = 24$ kN

$R_B = 72$ kN

$H = \frac{24 \cdot 12}{8} = 36$ kN

$(5+y)^2 + 6^2 = 13^2$

$y = 6{,}53$ m $\quad M_1 = 24 \cdot 6 - 36 \cdot 6{,}53 = -91$ kNm

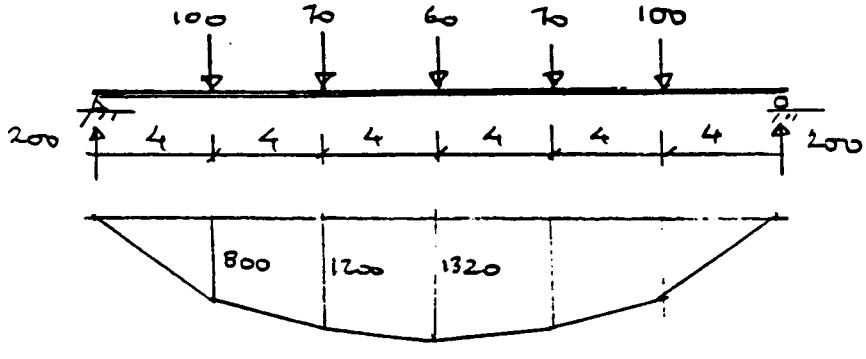
$M_2 = 72 \cdot 6 - 36 \cdot 6{,}53 - 8 \cdot 6 \cdot 3 = 53$ kNm

---

Ex 124

$R_A = R_B = 100 + 70 + \frac{60}{2} = 200$ kN

$H \cdot 10 - 100 \cdot 8 + 70 \cdot 4 - 200 \cdot 12 = 0 \qquad H = 132$ kN

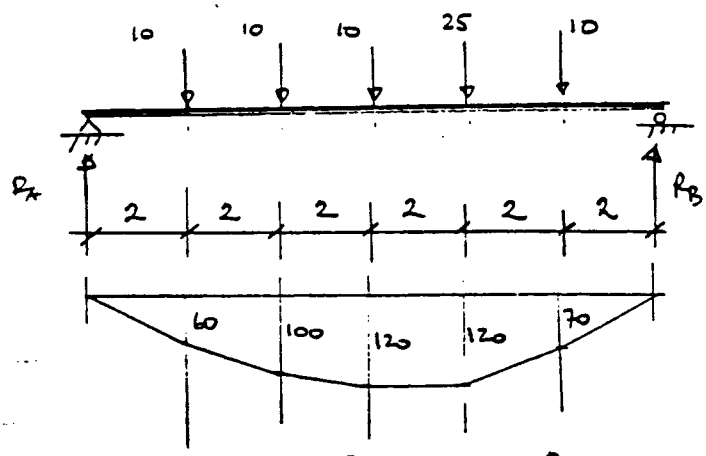


$\eta_1 = \frac{800}{132} = 6{,}06 \qquad \eta_2 = \frac{1200}{132} = 9{,}09 \qquad \eta_3 = 10{,}0$ m

---

Parabelbågen blir $y = \frac{4f(\ell x - x^2)}{\ell^2} = \frac{4 \cdot 3{,}6(12x - x^2)}{144}$

$$y = 0{,}1\,(12x - x^2)$$



$\circlearrowleft B$ $R_A \cdot 12 - 10 \cdot 10 - 10 \cdot 8 - 10 \cdot 6 - 25 \cdot 4 - 10 \cdot 2 = 0$

$R_A = 30$ kN

$R_B = 35$ kN

$H \cdot 3{,}6 + 10 \cdot 4 + 10 \cdot 2 - 30 \cdot 6 = 0$

$H = 33{,}3$ kN

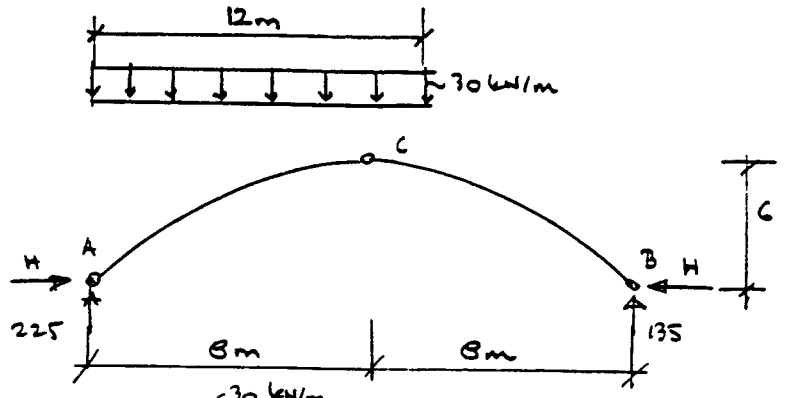
$\eta =$	0	1,8	3,0	3,6	3,6	2,1	0
$y =$	0	2,0	3,2	3,6	3,2	2,0	0
$M =$		-6,7	-6,7	0	+13,3	+3,3	0

$y' = 0{,}1\,(12 - 2x)$

$x = 8$ ger $y' = -0{,}4 = \tan\alpha$ $\quad \alpha = 21{,}8°$

$N = +33{,}3 \cdot \cos 21{,}8 + (35 - 10) \cdot \sin 21{,}8 = +40{,}2$ kN

Ex 126



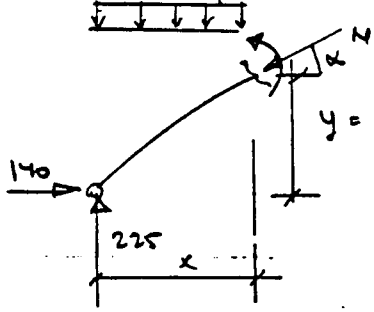
$\circlearrowright B$ $R_A \cdot 16 - 12 \cdot 30 \cdot 10 = 0$

$R_A = 225$ kN

$R_B = 135$ kN

$\circlearrowright C$ $225 \cdot 8 - 30 \cdot 8 \cdot 4 - H \cdot 6 = 0$

$H = 140$ kN



$$y = \frac{4f(lx - x^2)}{l^2} = \frac{4 \cdot 6(16x - x^2)}{256} = \frac{3}{32}(16x - x^2)$$

$$M_1 = 225 \cdot x - 140 \cdot \frac{3}{32}(16x - x^2) - \frac{30x^2}{2} =$$

$$= 225x + \frac{105}{8}x^2 + 210x - 15x^2$$

$$= 15x - \frac{15x^2}{8}$$

$x = 4$ $\quad y' = \frac{3}{32}(16 - 2x) = 0{,}75$

$\alpha = 36{,}9°$

$= (225 - 30 \cdot 4)\sin 36{,}87 + 140 \cdot \cos 36{,}87$

$= +175$ kN

$$\frac{dM}{dx} = 15 - \frac{30x}{8} = 0 \qquad x = 4{,}0\,m$$

$$M_{max} = 15 \cdot 4 - \frac{15 \cdot 4^2}{8} = 30\,kN$$

$$M_2 = 135 \cdot x_2 - 140 \cdot \frac{3}{32}(16x_2 - x_2^2) = -75x_2 + \frac{105x_2^2}{8}$$

$$\frac{dM_2}{dx} = -75 + \frac{210}{8}x_2 = 0 \qquad x_2 = 2{,}857\,m$$

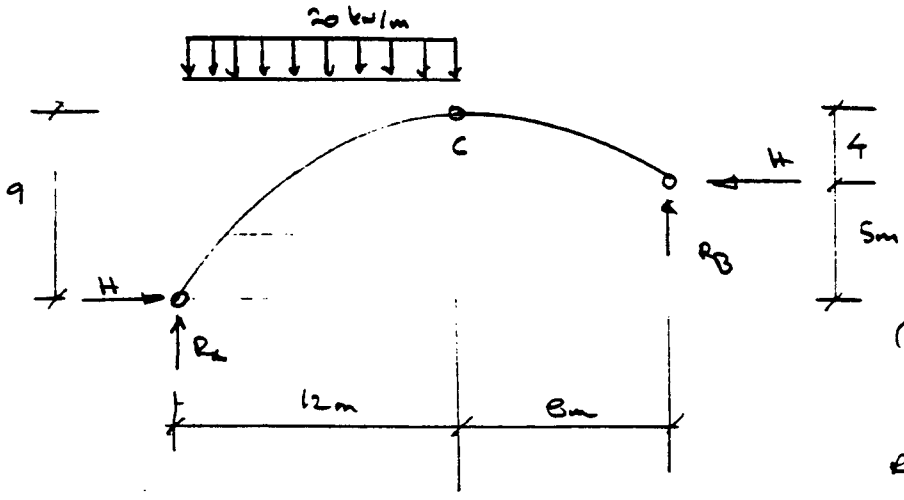
$$M_{max} = -75 \cdot 2{,}857 + \frac{105 \cdot 2{,}857^2}{8} = -107\,kNm$$

$$y' = \frac{3}{32}(16 - 2 \cdot 2{,}857) = 0{,}964$$

$$N = +135 \cdot \sin 43{,}96 + 140 \cos 43{,}96 = +194\,kN$$

$$\alpha = 43{,}96°$$

Ex 127



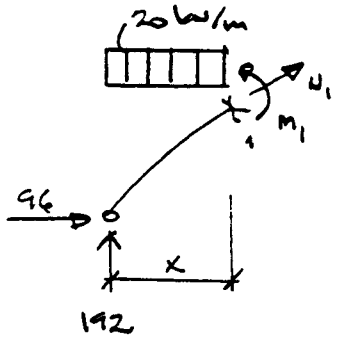
$\circlearrowleft A \quad R_B \cdot 20 - 20 \cdot 12 \cdot 6 + H \cdot 5 =$

$\curvearrowright \quad H \cdot 4 - R_B \cdot 8 = 0 \qquad H = 2R_B$

$R_B \cdot 20 + 10 R_B = 1440$

$\underline{\underline{R_B = 48 \text{ kN}}}$

$R_A = 12 \cdot 20 - 48 = \underline{\underline{192 \text{ kN}}}$



$$y = \frac{4 \cdot 9 \cdot (24x - x^2)}{24^2} = \frac{24x - x^2}{16}$$

$$M_1 = 192x - 96 \cdot \frac{24x - x^2}{16} - \frac{20x^2}{2} =$$

$$= 192x - 144x + 6x^2 - 10x^2 = 48x - 4x^2$$

$$\frac{dM}{dx} = 48 - 8x = 0 \qquad x = 6\text{m} \qquad M_1^+ = 48 \cdot 6 - 4 \cdot 6^2 = \underline{\underline{144 \text{ kNm}}}$$

$$\frac{dy}{dx} = \frac{24 - 2x}{16} = \frac{12}{16} \qquad \alpha_1 = 36{,}87$$

$$N_1 = +96 \cdot \cos 36{,}87 + 192 \cdot \sin 36{,}87 - 120 \cdot \sin 36{,}87 = \underline{\underline{+120 \text{ kN}}}$$

$$M_2 = 192x - 96 \cdot \frac{24x - x^2}{16} - 20 \cdot 12 \cdot (x - 6) =$$

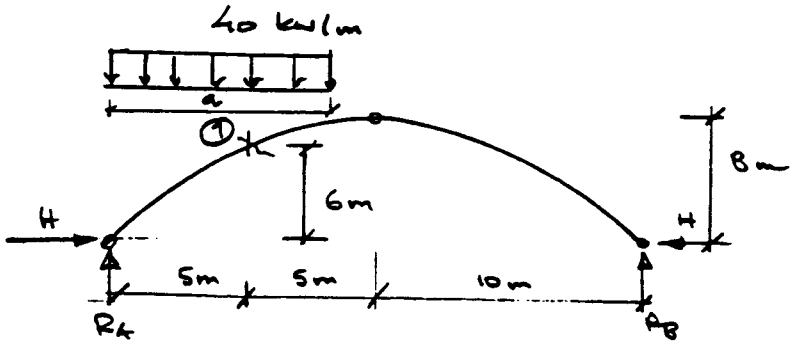
$$= 192x - 144x + 6x^2 - 240x + 1440$$

$$= 6x^2 - 192x + 1440$$

$$\frac{dM}{dx} = 12x - 192 = 0 \qquad x = 16\text{m} \qquad \frac{dy}{dx} = \frac{24 - 32}{16} \qquad \alpha_2 = 26{,}56$$

$$N_2 = +96 \cos 26{,}56 + 48 \cdot \sin 26{,}56 = \underline{\underline{+107 \text{ kN}}}$$

$$M_2^- = 6 \cdot 16^2 - 192 \cdot 16 + 1440 = \underline{\underline{-96 \text{ kNm}}}$$



Ex12B

$$y = \frac{4 \cdot f (l x - x^2)}{l^2} = \frac{4 \cdot 8 (20x - x^2)}{20^2} = 0{,}08 (20x - x^2)$$

$x = 5$ gir $y = 6$ m

$$\circlearrowright B \quad R_A \cdot 20 - 40 \cdot a \left(20 - \frac{a}{2}\right) = 0 \qquad \underline{R_A = 40a - a^2}$$

$$R_B = 40a - (40a - a^2) = \underline{a^2}$$

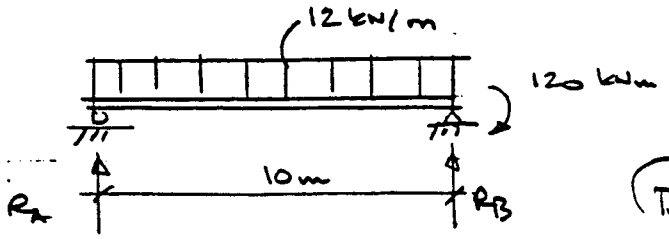
$$\circlearrowright C \quad H \cdot 8 - a^2 \cdot 10 = 0 \qquad H = \frac{10a^2}{8} = \underline{1{,}25 a^2}$$

$$M_1 = (40a - a^2) \cdot 5 - 1{,}25 a^2 \cdot 6 - 40 \cdot 5 \cdot 2{,}5 =$$
$$= 200a - 5a^2 - 7{,}5a^2 - 500$$
$$= 200a - 12{,}5a^2 - 500$$

$$\frac{dM_1}{da} = 200 - 25a = 0 \qquad \underline{a = 8 \text{ m}}$$

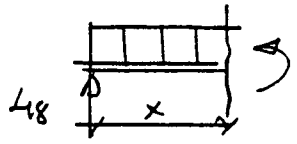
$$M_1 = 200 \cdot 8 - 12{,}5 \cdot 8^2 - 500 = \underline{300 \text{ kNm}}$$

Ex 129



$$\circlearrowleft B \quad R_A \cdot 10 + 120 - 12 \cdot 10 \cdot 5 = 0$$

$$R_A = 48 \text{ kN}$$



$$M = 48 \cdot x - 12x \cdot \frac{x}{2}$$

$$EIy'' = 6x^2 - 48x$$

$$EIy' = 2x^3 - 24x^2 + C$$

$$EIy = \frac{x^4}{2} - 8x^3 + Cx + D$$

$x = 0 \quad y = 0 \quad$ ger $\quad D = 0$

$x = 10 \quad y = 0 \quad$ ger $\quad \frac{10^4}{2} - 8000 + C \cdot 10 = 0$

$$\underline{C = 300}$$

$$\underline{EIy = \frac{x^4}{2} - 8x^3 + 300x} \qquad y = \frac{0{,}5x^4 - 8x^3 + 300x}{EI}$$

$x = 5$ ger $\quad y_{max} = \frac{0{,}5 \cdot 5^4 - 8 \cdot 5^3 + 300 \cdot 5}{EI} = \frac{812{,}5}{EI}$

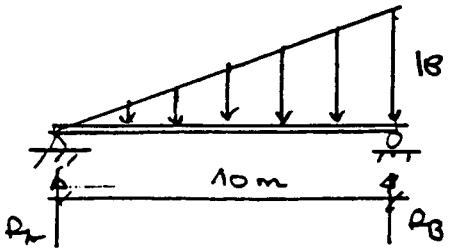
$$y_{max} = \frac{812{,}5}{29000} = 0{,}028 \text{ m} \quad \underline{= 28 \text{ mm}}$$

$$EIy' = 2x^3 - 24x^2 - 300$$

$x = 0$ ger $\quad \theta_A = \frac{300}{EI} = \frac{300}{29000} = \underline{0{,}0103 \text{ rad}}$

$x = 10$ ger $\quad \theta_B = \frac{2 \cdot 1000 - 2400 + 300}{29000} = \frac{-100}{29000} = \underline{-0{,}0034 \text{ rad}}$

Ex 130

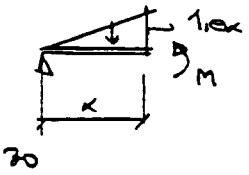


$$\frac{q_x}{x} = \frac{18}{10}$$

$$q_x = 1{,}8 \cdot x$$

$$R_A \cdot \cancel{10} - \frac{18 \cdot 10}{2} \cdot \frac{\cancel{10}}{3} = 0$$

$$R_A = 30 \text{ kN}$$



$$M = 30x - \frac{1{,}8x \cdot x}{2} \cdot \frac{x}{3} = 0$$

$$EIy'' = 0{,}3x^3 - 30x$$

$$EIy' = \frac{0{,}3x^4}{4} - 15x^2 + C$$

$$EIy = \frac{0{,}3x^5}{20} - 5x^3 + Cx + D$$

$x = 0 \quad y = 0 \qquad D = 0$

$x = 10 \quad y = 0 \qquad \frac{0{,}3 \cdot 10^5}{20} - 5000 + C \cdot 10 = 0$

$$C = 350$$

$$EIy = \frac{3x^5}{200} - 5x^3 + 350x$$

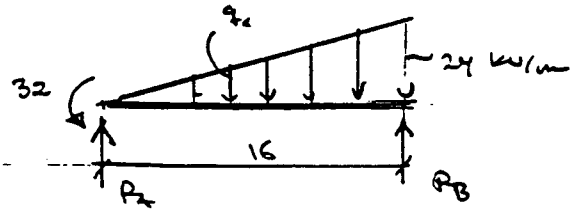
$$EI \cdot y_{max} = \frac{3 \cdot 5^5}{200} - 5 \cdot 5^3 + 350 \cdot 5 = 1172$$

$$y_{max} = \frac{1172}{31500} = 37{,}2 \text{ mm}$$

$EIy' = \frac{3x^4}{40} - 15x^2 + 350 \qquad x = 0$ qu $\theta_A = \frac{350}{31500} = 0{,}0111$ rad

$x = 10$ qu $\theta_B = \frac{3 \cdot 10^4}{40} - 1500 + 350 = -\frac{400}{31500} = -0{,}0127$ rad

Ex 131



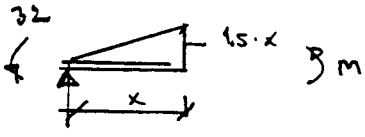
$$\frac{q_x}{x} = \frac{24}{16}$$

$$q_x = 1{,}5 \cdot x$$

$$\circlearrowleft B \quad R_A \cdot 16 - 32 - \frac{24 \cdot 16}{2} \cdot \frac{16}{3} = 0$$

$$R_A = 66\,kN$$

$$R_B = 126\,kN$$



$$M = 66 \cdot x - 32 - \frac{1{,}5 \cdot x \cdot x}{2} \cdot \frac{x}{3}$$

$$EIy'' = \frac{x^3}{4} - 66x + 32$$

$$EIy' = \frac{x^4}{16} - 33x^2 + 32x + C$$

$$EIy = \frac{x^5}{80} - 11x^3 + 16x^2 + Cx + D$$

$x = 0 \quad y = 0$ ger $D = 0$

$x = 16 \quad y = 0$ ger $\quad 0 = \frac{16^5}{80} - 11 \cdot 16^3 + 16 \cdot 16^2 + C \cdot 16$

$$C = 1740{,}8$$

$$EIy = \frac{x^5}{80} - 11x^3 + 16x^2 + 1740{,}8 \cdot x$$

$$EIy_{max} = \frac{8^5}{80} - 11 \cdot 8^3 + 16 \cdot 8^2 + 1740{,}8 \cdot 8 = 9728$$

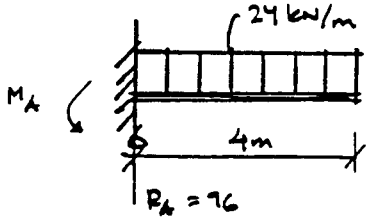
$$y_{max} = \frac{9728}{96000} = 0{,}101\ m = 101\ mm$$

$$EI\,y' = \frac{x^4}{16} - 33x^2 + 32x + 1741$$

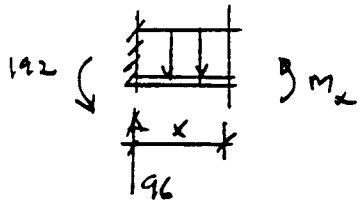
$$\theta_A = \frac{1741}{96000} = 0{,}0181\ rad$$

$$\theta_B = \frac{16^4}{16} - 33 \cdot 16^2 + 32 \cdot 16 + 1741 = -\frac{2099}{96000} = -0{,}0219\ rad$$

Ex 132



$M_A = 24 \cdot 4 \cdot 2 = 192$ kNm



$M_x = 96x - 192 - 24 \cdot x \cdot \frac{x}{2}$

$EIy'' = 12x^2 - 96x + 192$

$EIy' = 4x^3 - 48x^2 + 192x + C$

$EIy = x^4 - 16x^3 + 96x^2 + Cx + D$

$x = 0 \quad y = 0 \quad$ por $\quad D = 0$

$x = 0 \quad y' = 0 \quad$ por $\quad C = 0$

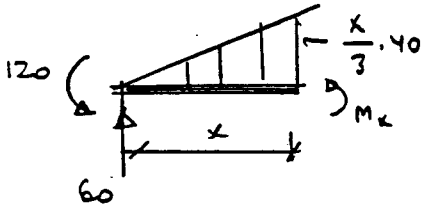
$$\underline{EIy = x^4 - 16x^3 + 96x^2}$$

$x = 4$ m por

$EIy^{max} = 4^4 - 16 \cdot 4^3 + 96 \cdot 4^2 = 768$

$$y_{max} = \frac{768}{31500} = \underline{24{,}4 \text{ mm}}$$

Ex 133



$M_x + 120 + \frac{x}{3} \cdot \frac{40x}{2} \cdot \frac{x}{3} - 60x = 0$

$EIy'' = \frac{20x^3}{9} - 60x + 120$

$EIy' = \frac{20x^4}{36} - 30x^2 + 120x + C$

$EIy = \frac{20x^5}{5 \cdot 36} - 10x^3 + 60x^2 + Cx + D$

$x = 0 \quad y = 0 \quad D = 0$

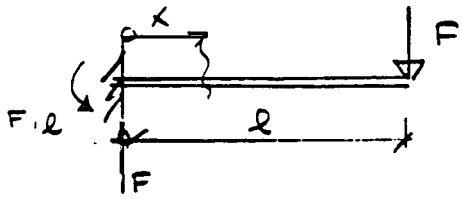
$x = 0 \quad y' = 0 \quad C = 0$

$$\underline{EIy = \frac{x^5}{9} - 10x^3 + 60x^2}$$

$x = 3$ por $\quad EIy^{max} = \frac{3^5}{9} - 10 \cdot 3^3 + 60 \cdot 3^2 = 297$

$$y^{max} = \frac{297}{16500} = 0{,}018 \text{ m} = \underline{18 \text{ mm}}$$

Ex 134



$$M_x = F\cdot x - F\cdot l$$

$$EIy'' = Fl - Fx$$

$$EIy' = Flx - \frac{Fx^2}{2} + C$$

$$EIy = \frac{Flx^2}{2} - \frac{Fx^3}{6} + Cx + D$$

$x=0 \quad y=0$ yr $\quad D=0$

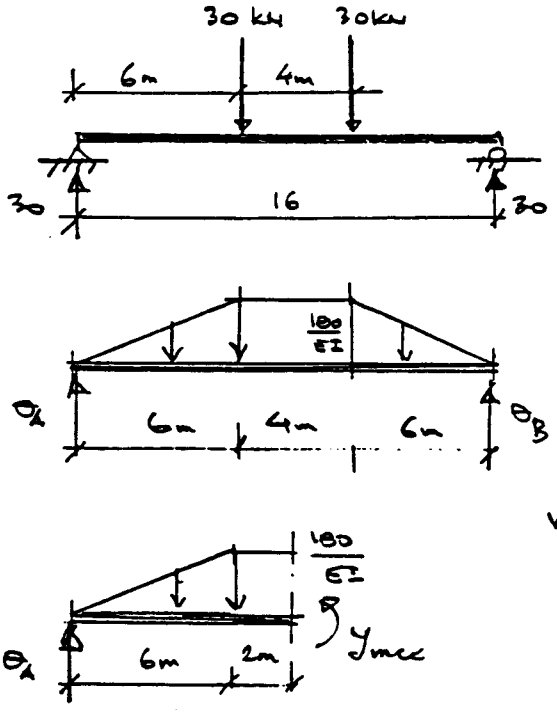
$x=0 \quad y'=0$ yr $\quad C=0$

$$EIy = \frac{F}{6}\left(3lx^2 - x^3\right)$$

$x=l$ yr

$$EIy = \frac{F}{6}\cdot\left(3l^3 - l^3\right) = \frac{Fl^3}{3}$$

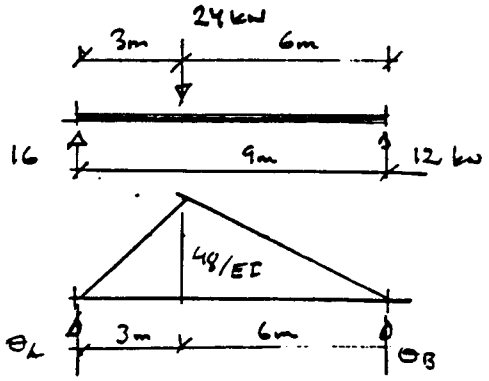
Ex 135



$$\theta_B = \frac{100}{EI}\cdot\left(2 + \frac{6}{2}\right) = \frac{900}{EI}$$

$$y_{max} = \frac{900}{EI}\cdot 0 - \frac{100}{EI}\cdot 2\cdot 1 - \frac{100}{EI}\cdot\frac{6}{2}\cdot 4 = 0$$

$$y_{max} = \frac{4600}{EI}$$



$M_{max} = 48$ kNm

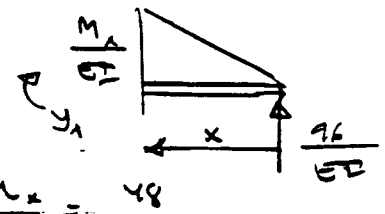
Ex 136

$$\circlearrowright B \quad \theta_A \cdot 9 - \frac{48}{EI}\cdot\frac{3}{2}\cdot 7 - \frac{48}{EI}\cdot\frac{6}{2}\cdot 4 = 0$$

$$\theta_A = \frac{120}{EI}$$

$$\circlearrowleft A \quad \theta_B \cdot 9 - \frac{48}{EI}\cdot\frac{3}{2}\cdot 2 - \frac{48}{EI}\cdot\frac{6}{2}\cdot 5 = 0$$

$$\theta_B = \frac{96}{EI}$$



$$\frac{M_x}{x} = \frac{48}{6}$$

$$M_x = 8 \cdot x$$

$$y_1 + \frac{M}{EI}\cdot\frac{x}{2}\cdot\frac{x}{3} - \frac{96}{EI}\cdot x = 0$$

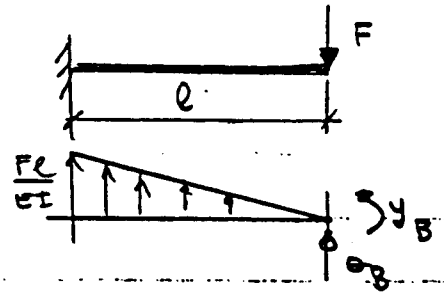
$$EI \cdot y_1 = 96x - \frac{8x^3}{6}$$

$$EI y_1' = 96 - 4x^2 = 0 \qquad x = \sqrt{24} = 4{,}90\,m$$

$$EI y_1 = 96\cdot\sqrt{24} - \frac{8\cdot 24\sqrt{24}}{6} = (96 - 32)\sqrt{24} = 64\sqrt{24} = 313{,}5$$

$$y_1 = \frac{313{,}5}{EI}$$

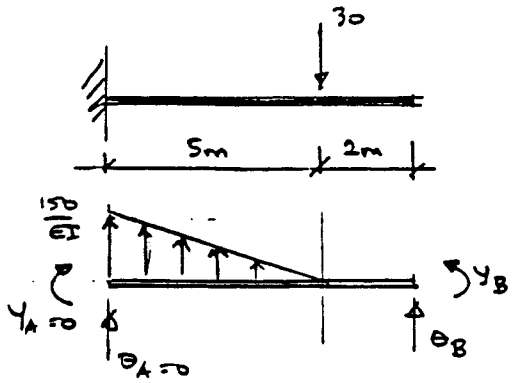
Ex 137



$$\uparrow \quad \theta_B + \frac{Fl}{EI}\cdot\frac{l}{2} = 0 \qquad \theta_B = -\frac{Fl^2}{2EI}$$

$$\curvearrowleft \quad y_B - \frac{Fl}{EI}\cdot\frac{l}{\cancel{2}}\cdot\frac{\cancel{2}l}{3} = 0 \qquad y_B = \frac{Fl^3}{3EI}$$

Ex 138

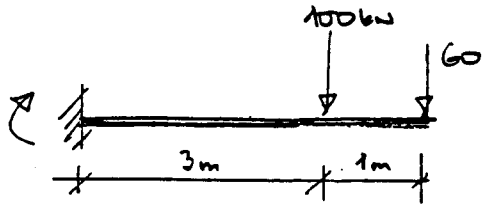


$$\theta_B = -\frac{150}{EI}\cdot\frac{5}{2} = -\frac{375}{EI} = -\frac{375}{25000} = -0{,}015 \text{ rad}$$

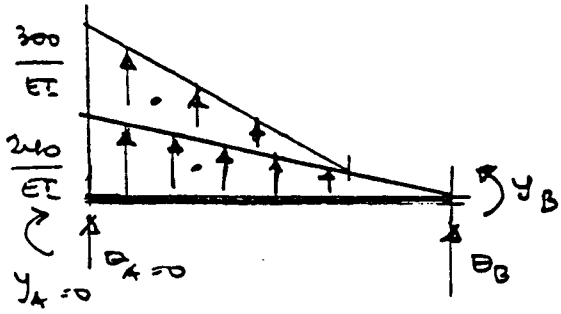
$$\circlearrowleft\ y_B - \frac{150}{EI}\cdot\frac{5}{2}\left(2+\frac{5\cdot 2}{3}\right) = 0$$

$$y_B = \frac{2000}{EI} = \frac{2000}{25000} = 0{,}080\,m = 80\,mm$$

Ex 139



$$M = -100\cdot 3 - 60\cdot 4 = -540\,kNm$$



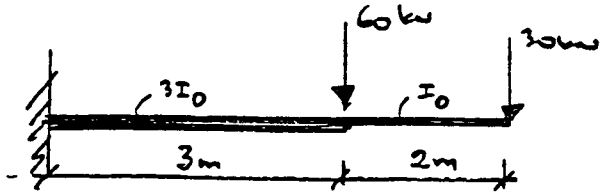
$$\uparrow\ \theta_B + \frac{300}{EI}\cdot\frac{3}{2} + \frac{240}{EI}\cdot\frac{4}{2} = 0$$

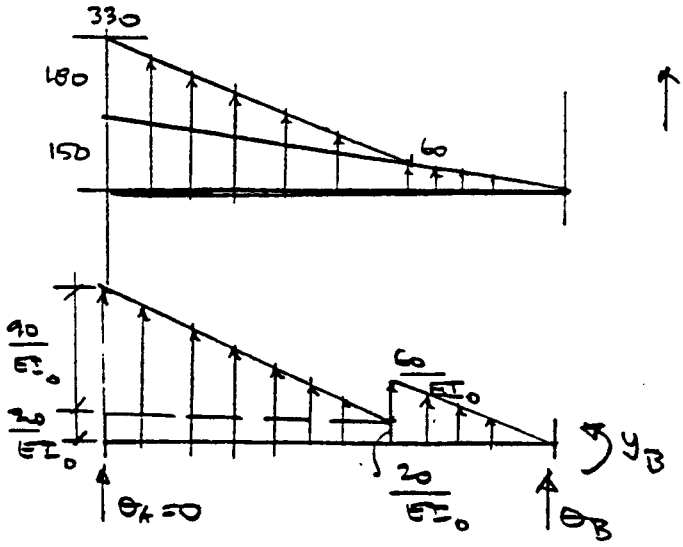
$$\theta_B = -\frac{930}{EI} = -\frac{930}{120000} = -0{,}0077 \text{ rad}$$

$$\circlearrowleft\ y_B - \frac{300}{EI}\cdot\frac{3}{2}\cdot 3 - \frac{240}{EI}\cdot\frac{4}{2}\cdot\frac{2\cdot 4}{3} = 0 \qquad y_B = \frac{2630}{EI}$$

$$y_B = \frac{2630}{120000} = 0{,}0219\,m$$

Ex 140





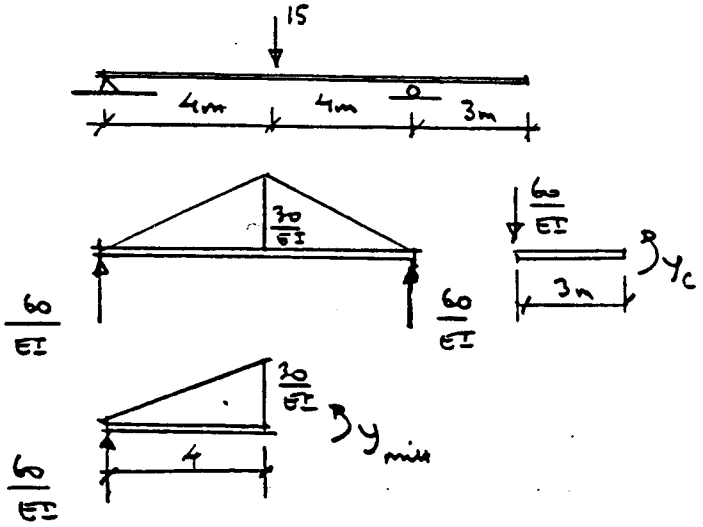
$$\uparrow \quad \theta_B + \frac{60}{EI_0}\cdot\frac{\cancel{2}}{\cancel{3}} + \frac{20}{EI_0}\cdot 3 + \frac{90}{EI_0}\cdot\frac{3}{2} = 0$$

$$\theta_B = -\frac{255}{EI_0}$$

$$\circlearrowleft \quad y_B - \frac{60}{EI_0}\cdot\frac{2\cdot 2}{3} - \frac{20\cdot 3}{EI_0}\cdot 3{,}5 - \frac{90\cdot 3}{EI_0\cdot 2}\cdot 4 = 0$$

$$y_B = \frac{80 + 210 + 540}{EI_0} = \frac{830}{EI_0}$$

Ex 141



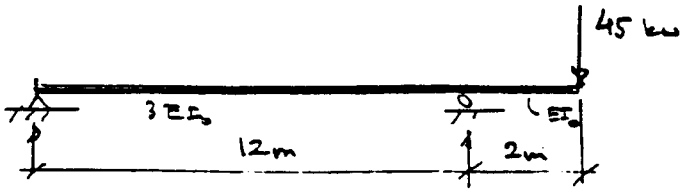
$$y_C + \frac{60}{EI}\cdot 3 = 0$$

$$y_C = -\frac{180}{EI}$$

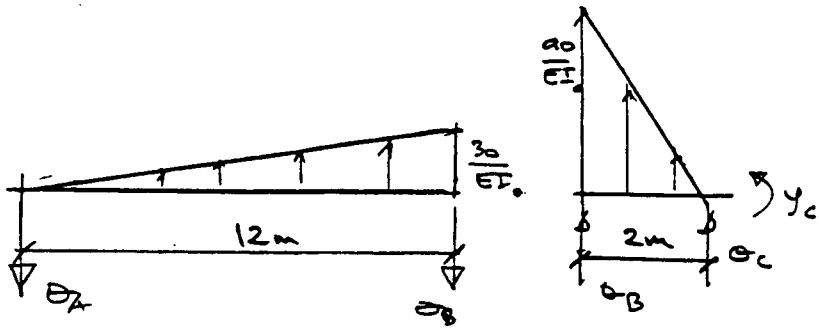
$$\circlearrowleft \quad y_{mid} - \frac{60}{EI}\cdot 4 + \frac{30}{EI}\cdot\frac{4}{2}\cdot\frac{4}{3} = 0$$

$$y_{mid} = \frac{160}{EI}$$

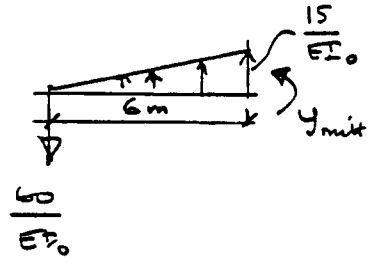
Ex 142



$M_B = -90$ kNm



$$\circlearrowright B \quad -\theta_A \cdot \cancel{12} + \frac{30}{EI_0} \cdot \frac{\cancel{12}}{2} \cdot 4 = 0 \qquad \theta_A = \frac{60}{EI_0} \qquad \theta_B = \frac{120}{EI_0}$$



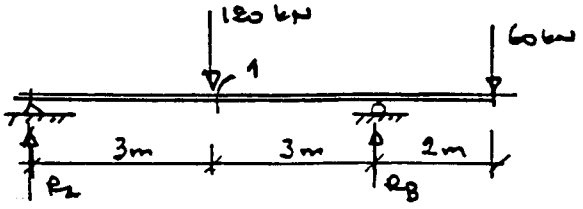
$$\circlearrowleft \quad y_{mitt} + \frac{60}{EI_0} \cdot 6 - \frac{15}{EI_0} \cdot \frac{6}{\cancel{2}} \cdot \cancel{2} = 0$$

$$y_{mitt} = -\frac{270}{EI_0}$$

$$\circlearrowleft \quad y_c - \frac{90}{EI_0} \cdot \frac{2}{\cancel{2}} \cdot \frac{\cancel{2} \cdot 2}{3} - \frac{120}{EI_0} \cdot 2 = 0$$

$$y_c = +\frac{360}{EI_0}$$

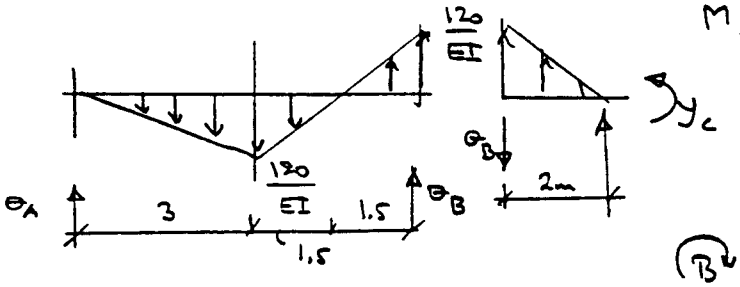
Ex 143



$$\circlearrowright B \quad R_A \cdot 6 - 120 \cdot 3 + 60 \cdot 2 = 0$$

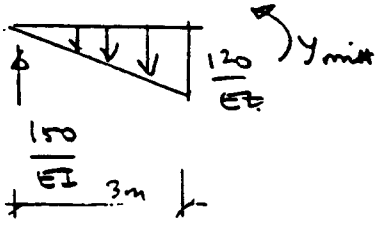
$$R_A = 40 \text{ kN}$$

$$M_1 = 40 \cdot 3 = 120 \text{ kNm}$$



$$\circlearrowright B \quad \theta_A \cdot 6 - \frac{120}{EI} \cdot \frac{3}{2} \cdot 4 - \frac{120}{EI} \cdot \frac{1.5}{2} \cdot (2.5 - 0.5) = 0$$

$$\theta_A = \frac{150}{EI}$$



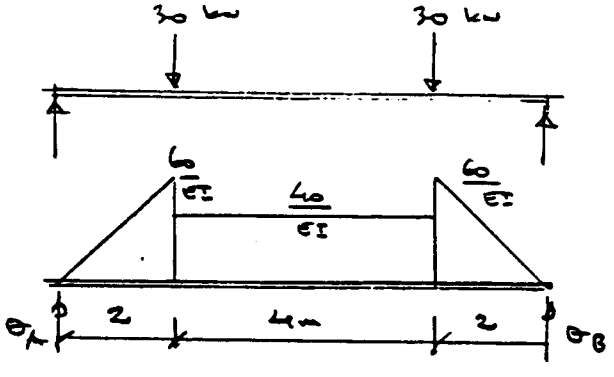
$$\theta_B = \frac{120}{EI} \cdot \frac{4.5}{2} - \frac{120}{EI} \cdot \frac{1.5}{2} - \frac{150}{EI}$$

$$\theta_B = \frac{30}{EI} \quad \text{(upwit)}$$

$$\circlearrowleft \quad y_{mid} + \frac{120}{EI} \cdot \frac{3}{2} \cdot 1 - \frac{150}{EI} \cdot 3 = 0 \qquad y_{mid} = \frac{270}{EI}$$

$$\circlearrowleft \quad y_c + \frac{30}{EI} \cdot 2 - \frac{120}{EI} \cdot \frac{\cancel{2}}{\cancel{2}} \cdot \frac{2 \cdot 2}{3} = 0 \qquad y_c = \frac{150}{EI}$$

Ex 144

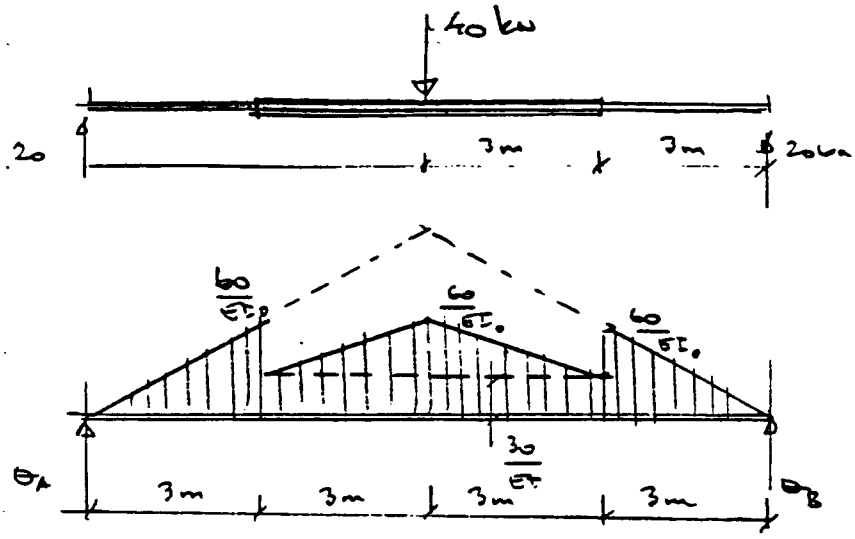


$$M = 30 \cdot 2 = 60$$

$$\theta_A = \theta_B = \frac{60}{EI} \cdot \frac{2}{2} + \frac{40}{EI} \cdot 2 = \frac{140}{EI}$$

$$y_{mid} = \frac{140}{EI} \cdot 4 - \frac{60}{EI} \cdot \frac{2}{2} \cdot \left(2 + \frac{2}{3}\right) - \frac{40}{EI} \cdot 2 \cdot 1 = \frac{320}{EI}$$

Ex 145



$$\theta_A = \theta_B = \frac{60}{EI_0}\cdot\frac{3}{2} + \frac{30}{EI_0}\cdot 3 + \frac{30\cdot 3}{EI_0\, 2} = \frac{225}{EI_0}$$

$$y_{mid} = \frac{225}{EI_0}\cdot 6 - \frac{60}{EI_0}\cdot\frac{3}{2}\cdot 4 - \frac{30}{EI}\cdot 3\cdot 1{,}5 - \frac{30}{EI}\cdot\frac{3}{2}\cdot 1 =$$

$$= \frac{1350 - 360 - 135 - 45}{EI_0} = \underline{\frac{810}{EI_0}}$$

Ex 146

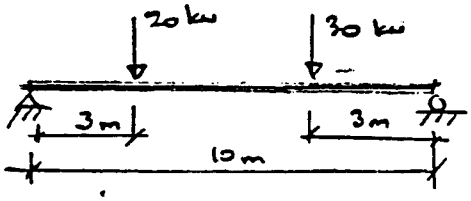
$$y_{mid} = \frac{5\cdot 30\cdot 8^4}{384\, EI} + \frac{40\cdot 3\,(3\cdot 8^2 - 4\cdot 9)}{48\, EI} = \frac{1990}{EI}$$

$$EI = 210\cdot 366{,}6 = 76990 \qquad y_{mid} = \frac{1990}{76990} = \underline{0{,}0258 \text{ m}}$$

$$\theta_A = \frac{8}{6EI}\left(\frac{30\cdot 8^2}{4} + \frac{40\cdot 3\cdot 5\,(3+8)}{64}\right) = \frac{777{,}5}{EI} = \frac{777{,}5}{76990} = \underline{0{,}010}$$

$$\theta_B = \frac{\quad}{\quad}\left(\frac{\quad}{\quad}(5+8)\right) = \frac{802{,}5}{EI} = \frac{802{,}5}{76990} = \underline{0{,}0104}$$

Ex 147



$$y_{mitt} = \frac{30\cdot 3\cdot(3\cdot 10^2 - 4\cdot 9)}{48\,EI} + \frac{20\cdot 3\,(3\cdot 10^2 - 4\cdot 9)}{48\,EI} = \frac{825}{EI}$$

$$EI = 11\,500\cdot\frac{90\cdot 720^3}{12}\cdot 10^{-9} = 30\,790 \text{ kNm}^2$$

$$y_{mitt} = \frac{825}{30\,790} = 0{,}0268 = \underline{26{,}8 \text{ mm}}$$

Ex 148

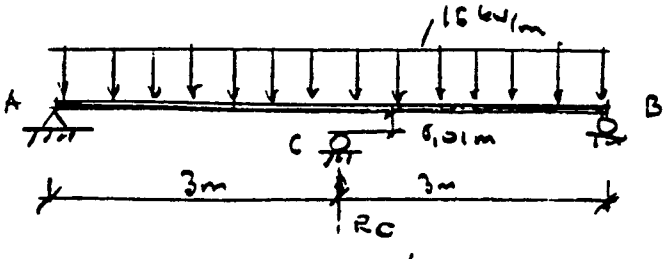
$$0{,}7^\circ = 0{,}7\,\frac{\pi}{180} = 0{,}0122 \text{ rad}$$

$$\theta_A = \theta_B = \frac{6}{6EI}\left(\frac{F\cdot 2\cdot 4\,(4+6)}{36} + \frac{F\cdot 2\cdot 4\,(2+6)}{36}\right) =$$

$$= \frac{4F}{EI} = 0{,}0122$$

$$F = \frac{0{,}122\,EI}{4} = \frac{210\,000\cdot 25{,}166\cdot 10^{6}}{10^{9}}\cdot\frac{0{,}122}{4} = \underline{164 \text{ kN}}$$

Ex 149



$$y_1 = \frac{5\cdot 16\cdot 6^4}{384\,EI} = \frac{270}{EI}$$

$$y_2 = \frac{R_c\cdot 6^3}{48\,EI} = \frac{4{,}5\,R_c}{EI}$$

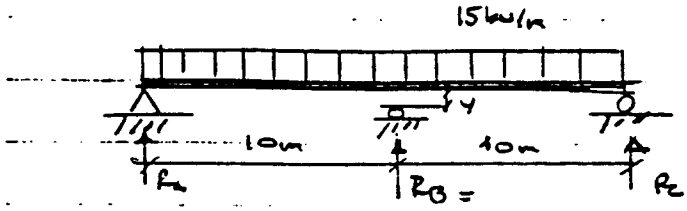
$$EI = \frac{210\,000\cdot 57{,}9\cdot 10^6}{10^9} = 210\cdot 57{,}9 = 12160 \text{ kNm}^2$$

$$\frac{270}{EI} - \frac{4{,}5\cdot R_c}{EI} = 0{,}01$$

$$270 - 4{,}5\,R_c = 0{,}01\cdot 12160 = 121{,}6$$

$$R_c = \frac{270 - 121{,}6}{4{,}5} = \underline{33{,}0 \text{ kN}}$$

Ex 150



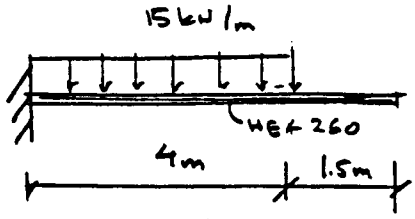
$$R_B = \frac{15 \cdot 20}{3} = 100\,kN$$

$$y = \frac{5 \cdot 15 \cdot 20^4}{384\,EI} - \frac{R_B \cdot 20^3}{48\,EI} = \frac{31250 - 16667}{EI} = \frac{14583}{EI}$$

$$EI = 210 \cdot 3591 = 754000 \qquad y = \frac{14583}{754000} = 0{,}0193$$

$$\underline{19{,}3\,mm}$$

Ex 151



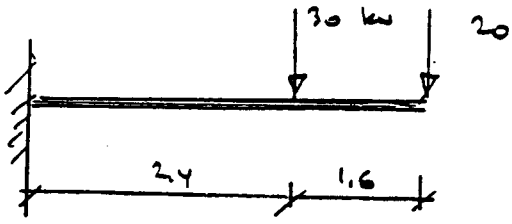
$$y_1 = \frac{15 \cdot 4^4}{8EI} = \frac{480}{EI} \qquad \varphi_1 = \frac{15 \cdot 4^3}{6EI} = \frac{160}{EI}$$

$$y_{max} = \frac{480}{EI} + 1{,}5 \cdot \frac{160}{EI} = \frac{720}{EI}$$

$$EI = 104{,}5 \cdot 210 = 21945\ kNm^2 = \frac{720}{21945} = \underline{32{,}8 \cdot 10^{-3}\,m}$$

$$= \underline{32{,}8\,mm}$$

Ex 152



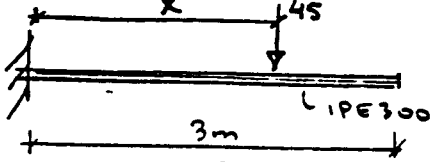
$$EI = 210 \cdot 1826 = 38346$$

$$y_1 = \frac{30 \cdot 2{,}4^3}{3EI} = \frac{138{,}2}{EI} \qquad \varphi_1 = \frac{30 \cdot 2{,}4^2}{2EI} = \frac{86{,}4}{EI}$$

$$y_{max} = \frac{138{,}2}{EI} + \frac{86{,}4}{EI} \cdot 1{,}6 + \frac{20 \cdot 4^3}{3EI} = \frac{703}{EI} = \frac{703}{38346} =$$

$$= \underline{18{,}3\,mm}$$

Ex 153



$EI = 210 \cdot 83{,}51 = 17550 \text{ kNm}^2$

$$y = \frac{45 \cdot x^3}{3EI} + \frac{45 \cdot x^2}{2EI}(3-x) = 0{,}012$$

$$15x^3 + 22{,}5x^2(3-x) = 0{,}012 \cdot 17550 = 210{,}6$$

$$15x^3 + 67{,}5x^2 - 22{,}5x^3 = 210{,}6$$

$$-7{,}5x^3 + 67{,}5x^2 = 210{,}6$$

$$-x^3 + 9x^2 = 28{,}1$$

$$x = \sqrt{\frac{x^3}{9} + 3{,}12} = 2{,}00$$

Prova $x = 2{,}00$

Ex 154

$$y = \frac{2F \cdot x^3}{3EI} + \frac{2Fx^2}{2EI}(l-x) - \frac{F \cdot l^3}{3EI} = 0$$

$$2x^3 + 3x^2(l-x) = l^3$$

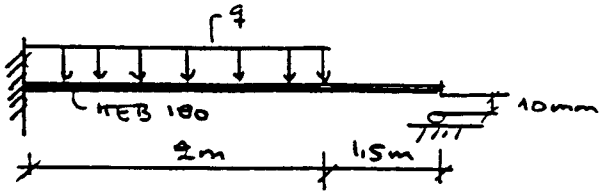
$$2x^3 + 3x^2 l - 3x^3 = l^3$$

$$-x^3 + 3x^2 l = l^3$$

$$x^2 = \frac{x^3 + l^3}{3l} \qquad x = \sqrt{\frac{x^3 + l^3}{3l}}$$

Prova $x = 0{,}653\,l$ que $x = 0{,}653\,l$

Ex 155



$$y = \frac{q \cdot 2^4}{8EI} + \frac{q \cdot 2^3}{6EI} \cdot 1{,}5 - \frac{10 \cdot 3{,}5^3}{3EI} = 0{,}01$$

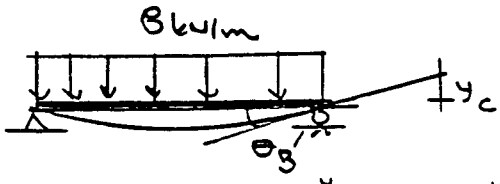
$$y = \frac{2q + 2q}{EI} - \frac{142{,}9}{EI} = 0{,}01$$

$$4q = 142{,}9 + 0{,}01 \cdot EI$$

$$EI = 210 \cdot 38{,}3 = 8043$$

$$4q = 142{,}9 + 80{,}4 \qquad q = 55{,}8 \text{ kN/m}$$

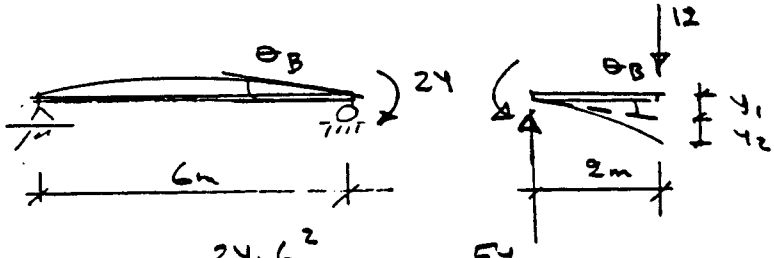
Ex 156



$$y_{mid} = \frac{5 \cdot 8 \cdot 6^4}{384\,EI} = \frac{135}{EI}$$

$$\theta_B = \frac{6}{6EI} \cdot \frac{8 \cdot 6^2}{4} = \frac{72}{EI} \qquad y_c = -\frac{72}{EI} \cdot 2 = -\frac{144}{EI}$$

Ex 157

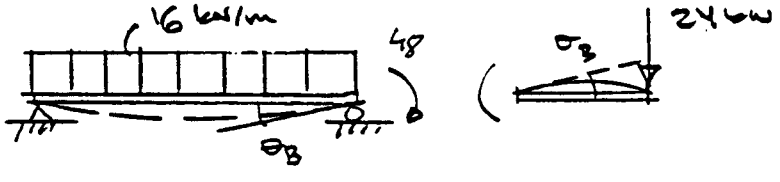


$$y_{mid} = -\frac{24 \cdot 6^2}{16\,EI} = -\frac{54}{EI}$$

$$\theta_B = -\frac{6}{6EI} \cdot 2 \cdot 24 = -\frac{48}{EI} \qquad y_c = \frac{48}{EI} \cdot 2 + \frac{12 \cdot 2^3}{3EI} =$$

$$= \frac{128}{EI}$$

Ex 158

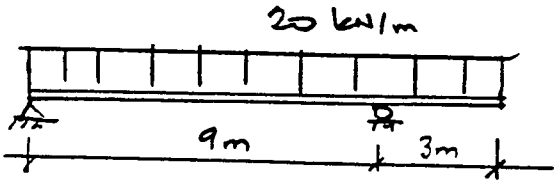


$$y_{mid} = \frac{5 \cdot 16 \cdot 6^4}{384\,EI} - \frac{48 \cdot 6^2}{16\,EI} = \frac{162}{EI}$$

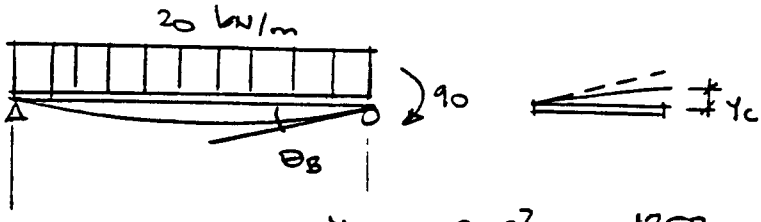
$$\theta_B = \frac{6}{6EI}\left(\frac{16 \cdot 6^2}{4} - 2 \cdot 48\right) = \frac{48}{EI}$$

$$y_c = -\frac{48}{EI} \cdot 2 + \frac{24 \cdot 2^3}{3EI} = -\frac{32}{EI}$$

Ex 159



$EI = 210 \cdot 1627 = 34167$



$$y_{max} = \frac{5 \cdot 20 \cdot 9^4}{384 EI} - \frac{90 \cdot 9^2}{16 EI} = \frac{1253}{EI} = \frac{1253}{34167} = \underline{36{,}7 \text{ mm}}$$

$$\theta_B = \frac{9}{6EI}\left(\frac{20 \cdot 9^2}{4} - 2 \cdot 90\right) = \frac{337{,}5}{EI}$$

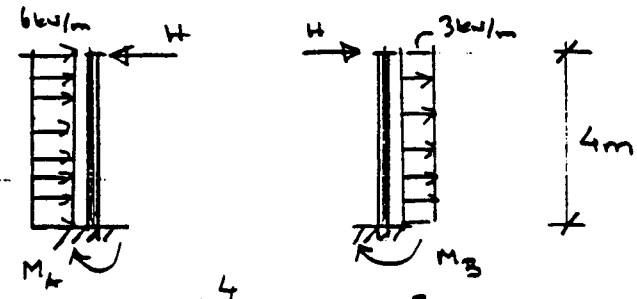
$$y_c = -\frac{337{,}5}{EI} \cdot 3 + \frac{20 \cdot 3^4}{8EI} = -\frac{810}{EI} = -\frac{810}{34167} = \underline{-23{,}7 \text{ mm}}$$

Ex 160

$$y = \frac{5 \cdot 30 \cdot 8^4}{384 EI} - \frac{R_B \cdot 8^3}{48 EI} = 0$$

$$2666{,}7 = 10{,}67 \, R_B \qquad R_B = \underline{250 \text{ kN}}$$

Ex 161



$$x_1 = \frac{6 \cdot 4^4}{8EI} - \frac{H \cdot 4^3}{3EI} = \frac{192}{EI} - \frac{64H}{3EI}$$

$$x_2 = \frac{3 \cdot 4^4}{8EI} + \frac{H \cdot 4^3}{3EI} = \frac{96}{EI} + \frac{64H}{3EI}$$

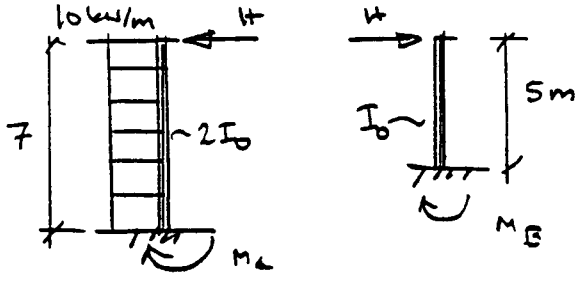
$$192 \cdot 3 - 64H = 96 \cdot 3 + 64H$$

$$288 = 128H \qquad \underline{H = 2{,}25 \text{ kN}}$$

$$M_A = 6 \cdot 4 \cdot 2 - 2{,}25 \cdot 4 = \underline{39 \text{ kNm}}$$

$$M_B = 3 \cdot 4 \cdot 2 + 2{,}25 \cdot 4 = \underline{33 \text{ kNm}}$$

Ex 162



$$x_1 = \frac{10 \cdot 7^4}{8EI_0} - \frac{H \cdot 7^3}{3EI_0} = \frac{1500{,}6 - 57{,}17H}{EI_0}$$

$$x_2 = \frac{H \cdot 5^3}{3EI_0} = \frac{41{,}67H}{EI_0}$$

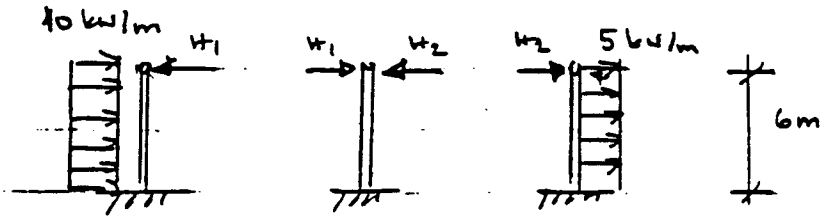
$$1500{,}6 - 57{,}17H = 41{,}67H$$

$$1500{,}6 = 98{,}84H \qquad \underline{H = 15{,}2 \text{ kN}}$$

$$M_A = 10 \cdot 7 \cdot 3{,}5 - 15{,}2 \cdot 7 = \underline{139 \text{ kNm}}$$

$$M_B = 15{,}2 \cdot 5 = \underline{76 \text{ kNm}}$$

Ex 163



$$x_1 = \frac{10\cdot 6^4}{8EI_0} - \frac{H_1\cdot 6^3}{3EI_0} = \frac{1620 - 72H_1}{EI_0}$$

$$x_2 = \frac{(H_1 - H_2)\cdot 6^3}{3EI_0\cdot 1.5} = \frac{48(H_1 - H_2)}{EI_0}$$

$$x_3 = \frac{H_2\cdot 6^3}{3EI_0} + \frac{5\cdot 6^4}{8EI_0} = \frac{72H_2 + 810}{EI_0}$$

$$1620 - 72H_1 = 48(H_1 - H_2) = 72H_2 + 810$$

FÖRSTA OCH SISTA GEK:

$$810 = 72(H_1 + H_2) \qquad \boxed{H_2 = 11{,}25 - H_1}$$

$$\frac{1620 - 72H_1}{48} = H_1 - 11{,}25 + H_1$$

$$33{,}75 = 1{,}5H_1 + H_1 + H_1 - 11{,}25$$

$$45 = 3{,}5H_1 \qquad \underline{H_1 = 12{,}86 \text{ kN}}$$

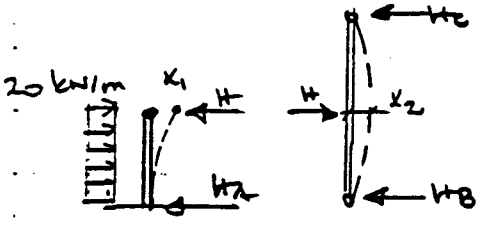
$$\underline{H_2 = -1{,}61 \text{ kN}}$$

$$M_A = 10\cdot 6\cdot 3 - 12{,}86\cdot 6 = \underline{103 \text{ kNm}}$$

$$M_B = (12{,}86 + 1{,}61)\cdot 6 = \underline{87 \text{ kNm}}$$

$$M_C = -1{,}61\cdot 6 + 5\cdot 6\cdot 3 = \underline{80 \text{ kNm}}$$

Ex 164



$$x_1 = \frac{20\cdot 4^4}{8EI_0} - \frac{H\cdot 4^3}{3EI_0} = \frac{640 - 21{,}33H}{EI_0}$$

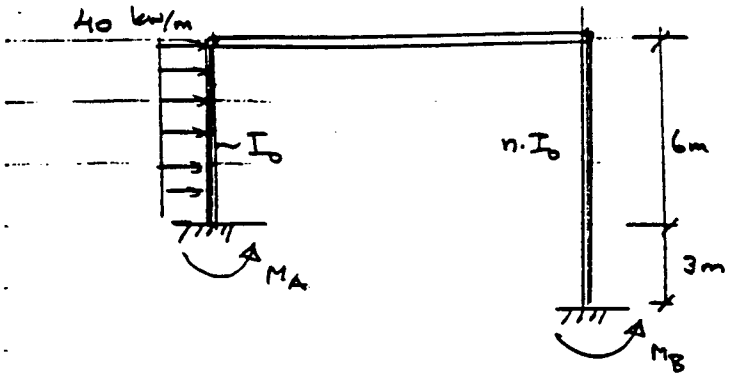
$$x_2 = \frac{H\cdot 8^3}{48\cdot 2EI_0} = \frac{5{,}33H}{EI_0}$$

$$640 = (21{,}33 + 5{,}33)H$$

$$H = 24 \text{ kN}$$

$$H_A = 20\cdot 4 - 24 = \underline{56 \text{ kN}}$$

$$\underline{H_B = H_C = 12 \text{ kN}}$$



$$K_1 = \frac{40 \cdot 6^4}{8EI_0} - \frac{H \cdot 6^3}{3EI_0} \qquad K_2 = \frac{H \cdot 9^3}{3EI_0 \cdot n}$$

$$\frac{6480}{\cancel{EI_0}} - \frac{72H}{\cancel{EI_0}} = \frac{243H}{\cancel{EI_0} \cdot n}$$

$$M_A = \frac{40 \cdot 6^2}{2} - H \cdot 6$$

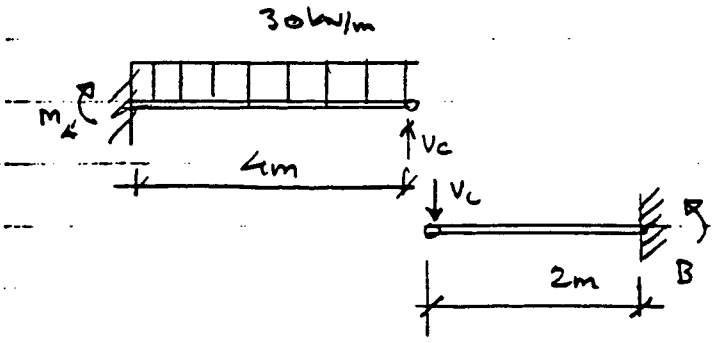
$$M_B = 9H \qquad M_A = M_B \text{ gir } 720 = 6H + 9H$$

$$H = 48 \text{ kN}$$

$$6480 - 72 \cdot 48 = \frac{243 \cdot 48}{n}$$

$$n = \frac{243 \cdot 48}{3024} = \underline{3{,}86}$$

Ex 166



$$y_1 = \frac{30 \cdot 4^4}{8EI} - \frac{V_C \cdot 4^3}{3EI} = \frac{960 - 21{,}33\, V_C}{EI}$$

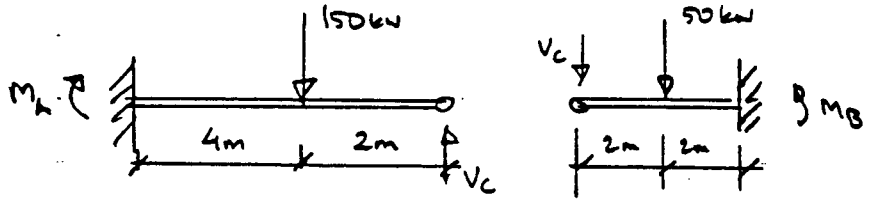
$$y_2 = \frac{V_C \cdot 2^3}{3EI} = \frac{2{,}67\, V_C}{EI}$$

$$960 - 21{,}33\, V_C = 2{,}67\, V_C \qquad \underline{V_C = 40 \text{ kN}}$$

$$M_A = 30 \cdot 4 \cdot 2 - 40 \cdot 4 = \underline{-80 \text{ kNm}}$$

$$M_B = 40 \cdot 2 = \underline{-80 \text{ kNm}}$$

Ex 167



$$y_1 = \frac{150 \cdot 4^3}{2 \cdot 3EI} + \frac{150 \cdot 4^2}{2 \cdot 2EI} \cdot 2 = \frac{2800}{EI}$$

$$y_2 = \frac{V_C \cdot 6^3}{2 \cdot 3EI} = \frac{36 V_C}{EI}$$

$$y_3 = \frac{V_C \cdot 4^3}{3EI} = \frac{21{,}33 V_C}{EI}$$

$$y_4 = \frac{50 \cdot 2^3}{3EI} + \frac{50 \cdot 2^2}{2EI} \cdot 2 = \frac{333{,}3}{EI}$$

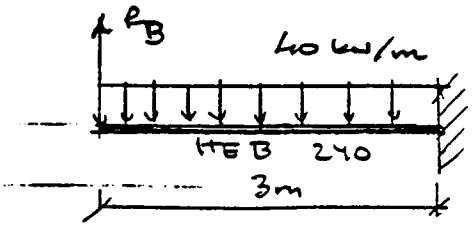
$$2800 - 36 V_C = 21{,}33 V_C + 333{,}3$$

$$V_C = 43{,}03 \text{ kN}$$

$$M_A = -150 \cdot 4 + 43 \cdot 6 = \underline{-342}$$

$$M_B = -43 \cdot 4 + 50 \cdot 2 = \underline{-272}$$

Ex 168



$$\frac{I}{A} = \frac{112{,}6 \cdot 10^6}{2 \cdot 113} = 0{,}50 \cdot 10^6$$

$$y_1 = \frac{40 \cdot 3^4}{8EI} - \frac{R_B \cdot 3^3}{3EI} = \frac{405 - 9R_B}{EI}$$

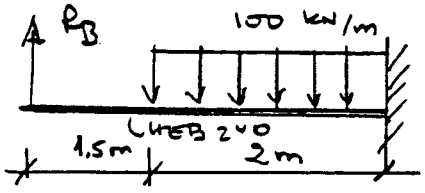
a) $405 - R_B \cdot 9 = 0$ $\quad$ $\underline{R_B = 45 \text{ kN}}$

b) $u = \dfrac{R_B \cdot 5}{EA}$ $\quad$ $405 - 9R_B = \dfrac{5R_B \cdot I}{A}$

$$405 = (9 + 5 \cdot 0{,}5)\, R_B$$

$$\underline{R_B = 35{,}2 \text{ kN}}$$

Ex 169



$$\frac{I}{A} = \frac{112{,}6 \cdot 10^6}{157} = 0{,}72 \cdot 10^6$$

$$y_1 = \frac{100 \cdot 2^4}{8EI} + \frac{100 \cdot 2^3}{6EI} \cdot 1{,}5 = \frac{400}{EI}$$

$$y_2 = \frac{R_B \cdot 3{,}5^3}{3EI} = \frac{14{,}3\, R_B}{EI}$$

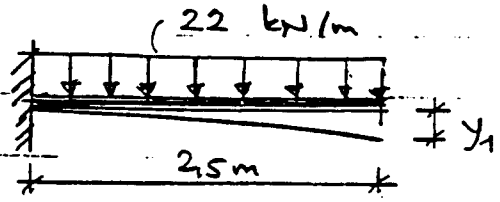
a) $400 - 14{,}3 \cdot R_B = 0$ $\quad$ $\underline{R_B = 29{,}0 \text{ kN}}$

b) $u = \dfrac{R_B \cdot 5}{EA}$ $\quad$ $400 - 14{,}3\, R_B = \dfrac{5 \cdot R_B \cdot I}{A} \cdot R_B$

$$400 = (14{,}3 + 5 \cdot 0{,}72)\, R_B$$

$$\underline{R_B = 22{,}3 \text{ kN}}$$

Ex 170

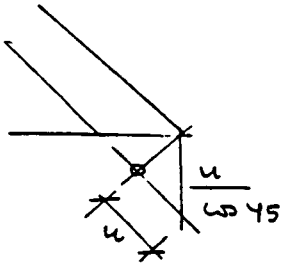


$$y_1 = \frac{22 \cdot 2{,}5^4}{8EI} = \frac{107{,}4}{EI}$$

$$y_2 = \frac{S \cdot \cos 45 \cdot 2{,}5^3}{3EI} = \frac{3{,}68 \cdot S}{EI}$$

a) $107{,}4 - 3{,}68 \cdot S = 0$ $\qquad$ $\underline{S = 29{,}2 \text{ kN}}$

b) $u = \dfrac{S \cdot 2{,}5}{\cos 45 \cdot EA} = \dfrac{3{,}5355}{EA} \cdot S$



$$y_3 = \frac{u}{\cos 45} = \frac{5{,}S}{EA}$$

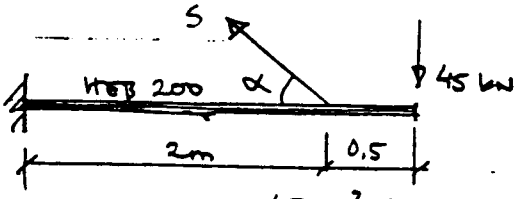
$$107{,}4 - 3{,}68S = \frac{5S \cdot I}{A}$$

$$\frac{I}{A} = \frac{19{,}43 \cdot 10^6}{157} = 0{,}124 \cdot 10^6$$

$$107{,}4 = (3{,}68 + 5 \cdot 0{,}124)\, S$$

$$\underline{S = 25{,}0 \text{ kN}}$$

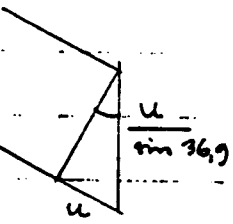
Ex 171



$\tan\alpha = \dfrac{1{,}5}{2}$ $\quad \alpha = 36{,}9°$

$$y_1 = \frac{45 \cdot 2^2}{6EI}(3 \cdot 2{,}5 - 2) = \frac{165}{EI} \qquad y_2 = \frac{S \sin 36{,}9 \cdot 2^3}{3EI} = \frac{1{,}60\,S}{EI}$$

a) $S = \dfrac{165}{1{,}60} = \underline{103 \text{ kN}}$ $\qquad$ b) $u = \dfrac{S \cdot 2}{\cos 36{,}9 \cdot EA} = \dfrac{2{,}5\,S}{EA}$

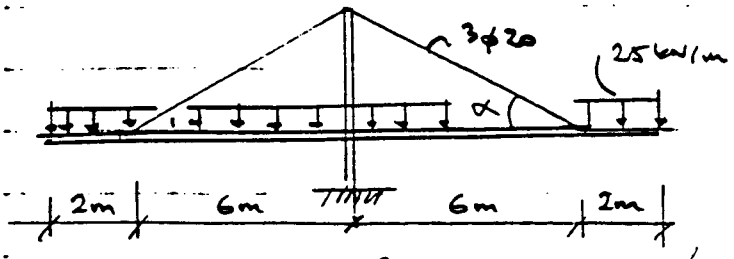


$$\frac{165 - 1{,}6\,S}{EI} = \frac{2{,}5 \cdot S}{EA \cdot \sin 36{,}9} \qquad \frac{I}{A} = \frac{57}{402} = 0{,}142$$

$$165 = S\left(1{,}6 + 4{,}16 \cdot \frac{I}{A}\right) = S\,(1{,}6 + 4{,}16 \cdot 0{,}142)$$

$$\underline{S = 75{,}3 \text{ kN}}$$

Ex 172



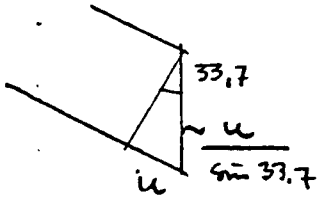
$$\tan\alpha = \frac{4}{6}$$

$$\alpha = 33{,}7^\circ$$

$$y_1 = \frac{25\cdot 6^2}{24EI}\left(6\cdot 8^2 - 4\cdot 8\cdot 6 + 6^2\right) = \frac{8550}{EI}$$

$$y_2 = \frac{S\cdot \sin 33{,}7\cdot 6^3}{3EI} = \frac{39{,}95\cdot S}{EI}$$

$$u = \frac{S\cdot 6}{\cos 33{,}7\cdot EA} = \frac{7{,}212\cdot S}{EA} \qquad y_3 = \frac{u}{\sin 33{,}7} = \frac{13\cdot S}{EA}$$



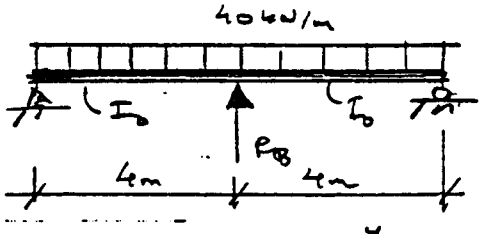
$$8550 - 39{,}95\,S = 13S\cdot\frac{I}{A}$$

$$\frac{I}{A} = \frac{576{,}8}{942} = 0{,}612$$

$$8550 = \left(39{,}95 + 13\cdot 0{,}612\right)\cdot S$$

$$S = 178 \text{ kN}$$

Ex 173



$$y_1 = \frac{5\cdot 40\cdot 8^4}{384EI_0} - \frac{R_B\cdot 8^3}{48EI} = \frac{2133 - 10{,}67\cdot R_B}{EI_0}$$

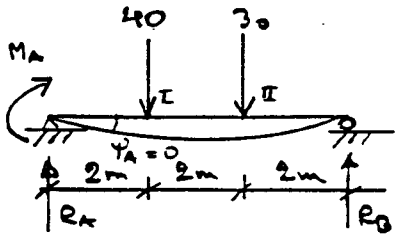
$$y_2 = \frac{R_B\cdot 10^3}{48EI\cdot 5} = \frac{20{,}83\,R_B}{5\cdot EI_0} = \frac{4{,}17\,R_B}{EI_0}$$

$$2133 - 10{,}67\,R_B = 4{,}17\,R_B$$

$$2133 = 14{,}83\,R_B \qquad R_B = 144 \text{ kN}$$

$$R_A = R_C = \frac{40\cdot 8 - 144}{2} = 88 \text{ kN}$$

Ex 174



$$\varphi_A = \frac{L}{6EI}\left(\frac{40\cdot 2\cdot 4\cdot 10}{36} + \frac{30\cdot 4\cdot 2\cdot 8}{36} + 2M_A\right) = 0$$

$$M_A = -71 \text{ kNm}$$

$$M_{II} = 21.5\cdot 2 = 43 \text{ kNm}$$

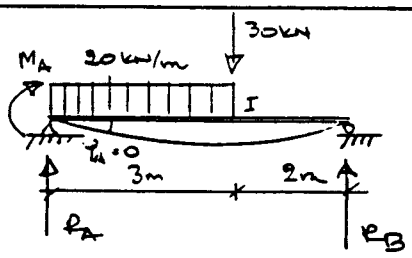
$$M_I = 21.5\cdot 4 - 30\cdot 2 = 26 \text{ kNm}$$

$$\circlearrowright A \quad -71 + 40\cdot 2 + 30\cdot 4 - R_B\cdot 6 = 0$$

$$\circlearrowright B \quad R_A\cdot 6 - 30\cdot 2 - 40\cdot 4 - 71 = 0$$

$$R_A = 21{,}5 \text{ kN} \qquad R_B = 48{,}5 \text{ kN}$$

Ex 175



$$\varphi_A = \frac{L}{6EI}\left(\frac{30\cdot 3\cdot 2\cdot 7}{25} + \frac{20\cdot 9}{25}(5-1.5)^2 + 2M_A\right) = 0$$

$$M_A = -69{,}3 \text{ kNm}$$

$$\circlearrowright A \quad -69.3 + 20\cdot 3\cdot 1.5 + 30\cdot 3 - R_B\cdot 5 = 0$$

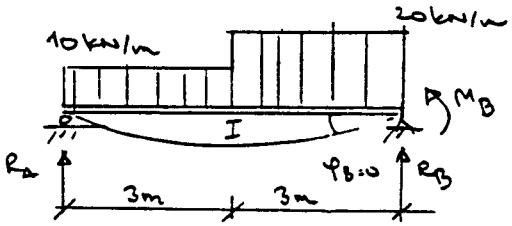
$$\circlearrowright B \quad -69.3 - R_A\cdot 5 - 20\cdot 3\cdot 3.5 - 30\cdot 2 = 0$$

$$R_A = 67{,}9 \text{ kN} \qquad R_B = 22{,}1 \text{ kN}$$

$$M_I = 22.1\cdot 2 = 44{,}2 \text{ kNm}$$

$$M^0_{AI} = \frac{20\cdot 3^2}{8} = 22{,}5 \text{ kNm}$$

Ex 176



$$\varphi_B = \frac{L}{6EI}\left(\frac{10\cdot 6^2}{4} + \frac{10\cdot 9}{36}(6-1.5)^2 + 2M_B\right) = 0$$

$$M_B = -70{,}3 \text{ kNm}$$

$$\circlearrowright A \quad 10\cdot 3\cdot 1.5 + 20\cdot 3\cdot 4.5 + 70{,}3 - R_B\cdot 6 = 0$$

$$\circlearrowright B \quad R_A\cdot 6 - 10\cdot 3\cdot 4.5 - 20\cdot 3\cdot 1.5 + 70{,}3 = 0$$

$$R_A = 25{,}8 \text{ kN} \qquad R_B = 64.2 \text{ kN}$$

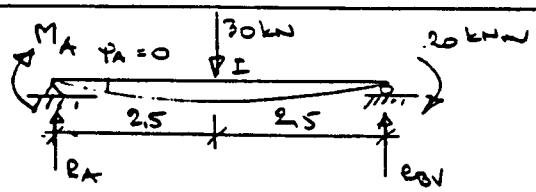
$$M^{max}_{AB} = \frac{25.8^2}{20} = 33{,}3 \text{ kNm}$$

$$M_I = 25.8\cdot 3 - 10\cdot 3\cdot 1.5 = 32{,}4$$

$$M^0_{AI} = \frac{10\cdot 9}{8} = 11{,}3 \text{ kNm}$$

$$M^0_{IB} = \frac{20\cdot 9}{8} = 22{,}5 \text{ kNm}$$

Ex 177



$$\varphi_A = \frac{L}{6EI}\left(\frac{3\cdot 30\cdot 5}{8} + 2M_A - 20\right) = 0$$

$$M_A = -18{,}1 \text{ kNm}$$

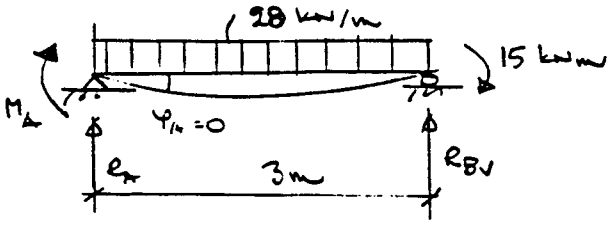
$$\circlearrowright A \quad -18.1 + 30\cdot 2.5 + 20 - R_{BV}\cdot 5 = 0$$

$$\circlearrowright B \quad -18.1 - 30\cdot 2.5 + 20 + R_A\cdot 5 = 0$$

$$R_A = 14{,}6 \text{ kN} \qquad R_{BV} = 15.4 \text{ kN}$$

$$M_I = 15.4\cdot 2.5 - 20 = 18.5 \text{ kNm}$$

Ex 178



$$\varphi_A = \frac{L}{6EI}\left(\frac{28\cdot 3^2}{4} + 2M_A - 15\right) = 0$$

$$\underline{M_A = -24 \text{ kNm}}$$

$$\circlearrowleft B \quad -24 + R_A \cdot 3 - 28\cdot 3\cdot 1{,}5 + 15 = 0$$

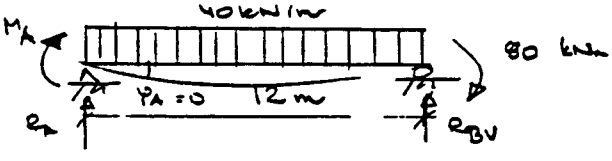
$$\circlearrowleft A \quad -24 + 15 + 28\cdot 3\cdot 1{,}5 - R_{BV}\cdot 3 = 0$$

$$\underline{R_A = 45 \text{ kN}} \qquad \underline{R_{BV} = 39 \text{ kN}}$$

$$M_{AB}^{max} = \frac{45^2}{2\cdot 28} - 24 = \underline{12{,}2 \text{ kNm}}$$

$$M_{AB}^{o} = \frac{28\cdot 9}{8} = \underline{31{,}5 \text{ kNm}}$$

Ex 179



$$\varphi_A = \frac{L}{6EI}\left(\frac{40\cdot 144}{4} + 2M_A - 80\right) = 0$$

$$\underline{M_A = -680 \text{ kNm}}$$

$$\circlearrowleft B \quad R_A\, 12 - 680 - 40\cdot 12\cdot 6 - 80 = 0$$

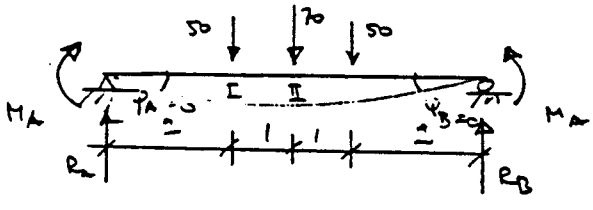
$$\circlearrowleft A \quad -680 + 40\, 12\cdot 6 + 80 - R_{BV}\cdot 12 = 0$$

$$\underline{R_A = 290 \text{ kN}} \qquad \underline{R_B = 190 \text{ kN}}$$

$$M_{AB}^{max} = \frac{190^2}{2\cdot 40} - 80 = \underline{374 \text{ kNm}}$$

$$M_{AB}^{o} = \frac{40\cdot 12^2}{8} = \underline{720 \text{ kNm}}$$

Ex 180



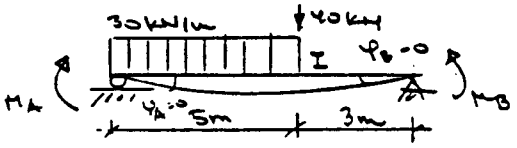
$$\varphi_A = \frac{L}{6EI}\left(\frac{3\cdot 70\cdot 6}{8} + \frac{50\cdot 2\cdot 4\cdot 10}{36} + \frac{50\cdot 4\cdot 2\cdot 8}{36} + 3M_A\right) = 0$$

$$\underline{M_A = -119 \text{ kNm}}$$

$$R_A = R_B = (100 + 70) : 2 = \underline{85 \text{ kN}}$$

$$M_I = 85\cdot 2 - 119 = \underline{51 \text{ kNm}}$$

$$M_{II} = 85\cdot 3 - 119 - 50\cdot 1 = \underline{86 \text{ kNm}}$$



$$\varphi_A = \frac{L}{6EI}\left(\frac{30\cdot 25}{64}(8-2.5)^2 + \frac{40\cdot 5\cdot 3\cdot 11}{64} + 2M_A + M_B\right) = 0$$

$$\varphi_B = \frac{L}{6EI}\left(\frac{30\cdot 25}{128}(64-12.5) + \frac{40\cdot 5\cdot 3\cdot 13}{64} + 2M_B + M_A\right) = 0$$

$$\begin{cases} 457.6 + 2M_A + M_B = 0 \\ 423.6 + 2M_B + M_A = 0 \end{cases}$$

$$M_B = -2M_A - 457.6$$

$$423.6 - 4M_A - 915.2 + M_A = 0$$

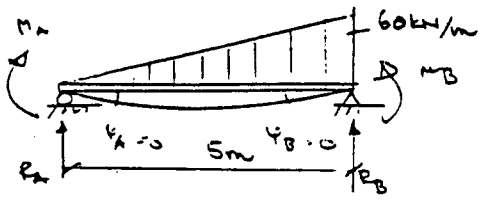
$$M_A = \underline{-163.9 \text{ kNm}} \qquad \underline{M_B = -129.9 \text{ kNm}}$$

$$R_A = \frac{3}{8}\cdot 40 + \frac{30\cdot 5\cdot 5.5}{8} + \frac{164-130}{8} = 122 \text{ kN}$$

$$R_B = \frac{5}{8}\,40 + \frac{30\cdot 5\cdot 2.5}{8} + \frac{130-164}{8} = \underline{67.6 \text{ kN}}$$

$$M_{AB}^{max} = \frac{122.4^2}{60} - 164 = \underline{86 \text{ kNm}} \qquad M_{AB}^{0} = \frac{30\cdot 25}{8} = \underline{94 \text{ kNm}}$$

$$M_I = 67.6\cdot 3 - 130 = \underline{73 \text{ kNm}}$$



$$\varphi_A = \frac{L}{6EI}\left(\frac{7\cdot 60\cdot 25}{60} + 2M_A + M_B\right) = 0$$

$$\varphi_B = \frac{L}{6EI}\left(\frac{8\cdot 60\cdot 25}{60} + 2M_B + M_A\right) = 0$$

$$\begin{cases} 175 + 2M_A + M_B = 0 \\ 200 + 2M_B + M_A = 0 \end{cases}$$

$$M_A = -2M_B - 200$$

$$175 - 4M_B - 400 + M_B = 0$$

$$\underline{M_B = -75 \text{ kNm}}$$

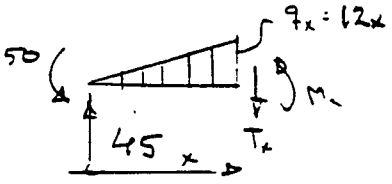
$$M_A = 150 - 200 = \underline{-50 \text{ kNm}}$$

$$R_A = \frac{60\cdot 5}{2}\cdot\frac{1}{3} + \frac{50-75}{5} = \underline{45 \text{ kN}}$$

$$R_B = \frac{60\cdot 5}{2}\cdot\frac{2}{3} + \frac{75-50}{5} = \underline{105 \text{ kN}}$$

$$\frac{q_x}{60} = \frac{x}{5}$$

$$\underline{q_x = 12x}$$



$$\downarrow \quad V_x + \frac{12x\cdot x}{2} - 45 = 0$$

$$\circlearrowleft \quad M_x + \frac{12x\cdot x}{2}\cdot\frac{x}{3} - 45\cdot x + 50 = 0$$

$$\underline{V_x = 45 - 6x^2}$$

$$\underline{M_x = 45x - 2x^3 - 50}$$

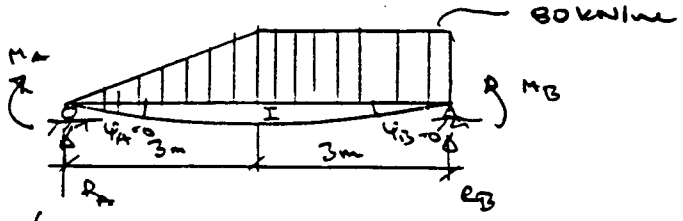
$V_x = 0$ ger

$$x^2 = \frac{45}{6} = 7{,}5 \text{ m}$$

$$x = 2{,}74 \text{ m}$$

x = 1	V = 39	M = -7,0
x = 2	V = 21	M = +24
x = 3	V = -9	M = +31
x = 4	V = -51	M = +2
x = 5	V = -105	M = -75

$$M_{AB}^{max} = \underline{32{,}2 \text{ kNm}}$$



$$\varphi_A = \frac{L}{6EI}\left(\frac{40\cdot 36}{4} - \frac{40\cdot 9}{60\cdot 36}(20\cdot 36 - 15\cdot 18 + 3\cdot 9) + 2M_A + M_B\right) = 0$$

$$\varphi_B = \frac{L}{6EI}\left(\frac{40\cdot 36}{4} - \frac{40\cdot 9}{60\cdot 36}(10\cdot 36 - 3\cdot 9) + 2M_B + M_A\right) = 0$$

$$\begin{cases} 360 - 79.5 + 2M_A + M_B = 0 \\ 360 - 55.5 + 2M_B + M_A = 0 \end{cases}$$

$$M_B = -2M_A - 280.5$$

$$304.5 - 4M_A - 561 + M_A = 0$$

$$\left(\underline{M_A = -85.5} \qquad \underline{M_B = -109.5}\right)$$

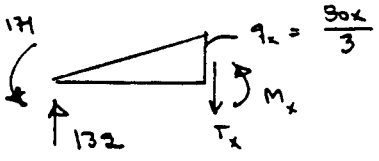
Fördubbling av lasten ger $\underline{M_A = -171 \text{ kNm}}$ $\underline{M_B = -219 \text{ kNm}}$

$$\circlearrowright B \quad R_A\cdot 6 - \frac{3\cdot 80}{2}\cdot 4 - 3\cdot 80\cdot 1.5 + 219 - 171 = 0$$

$$\circlearrowleft A \quad R_B\cdot 6 - \frac{3\cdot 80}{2}\cdot 2 - 3\cdot 80\cdot 4.5 - 219 + 171 = 0$$

$$\underline{R_A = 132 \text{ kN}} \qquad \underline{R_B = 228 \text{ kN}}$$

$$M_I = 228\cdot 3 - 219 - 80\cdot 3\cdot 1.5 = \underline{105 \text{ kNm}} \qquad \frac{q_x}{x} = \frac{80}{3}$$



$$\downarrow \quad T_x + \frac{80x}{3}\cdot\frac{x}{2} - 132 = 0$$

$$\circlearrowleft \quad M_x + \frac{80x}{3}\cdot\frac{x}{2}\cdot\frac{x}{3} + 171 - 132x = 0$$

$$\underline{T_x = 132 - \frac{40x^2}{3}}$$

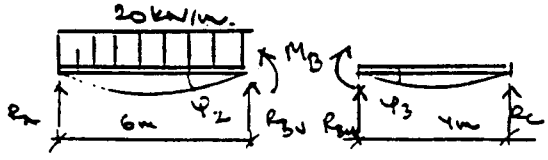
$$\underline{M_x = 132x - \frac{40x^3}{9} - 171}$$

$$M_{AB}^{max} = \frac{228^2}{160} - 219 = \underline{105.9 \text{ kNm}}$$

$$M_{AB}^{0} = \frac{80\cdot 9}{8} = \underline{90 \text{ kNm}}$$

$x = 1$	$T = 118.7$ kN	$M = -43.4$
$x = 2$	$T = 78.7$ kN	$M = +57.4$
$x = 3$	$T = 12$	$M = +105$

Ex 184



$\varphi_2 = -\varphi_3$

$$\frac{\cancel{6}\,3}{\cancel{6EI}}\left(\frac{20\cdot 6^2}{4}+2M_B\right)+\frac{4}{\cancel{6EI}}\cdot 2M_B=0$$

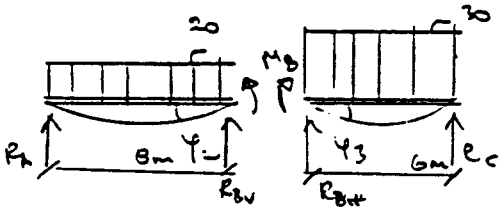
$$540+6M_B+4M_B=0 \qquad \underline{M_B=-54\ \text{kNm}}$$

$$R_A=\frac{20\cdot 6}{2}-\frac{54}{6}=\underline{51\ \text{kN}} \qquad M_{AB}^{max}=\frac{51^2}{40}=\underline{65\ \text{kNm}}$$

$$R_{BV}=120-51=\underline{69\ \text{kN}}$$

$$R_{BH}=+\frac{54}{4}=\underline{13{,}5\ \text{kN}} \qquad M^0_{AB}=\frac{20\ 36}{8}=\underline{90\ \text{kNm}}$$

Ex 185



$\varphi_2 = -\varphi_3$

$$\frac{\cancel{8}^{\,4}}{\cancel{6EI}}\left(\frac{20\ 64}{4}+2M_B\right)+\frac{\cancel{6}^{\,3}}{\cancel{6EI}}\left(\frac{20\cdot 36}{4}+2M_B\right)=0$$

$$1280+8M_B+810+6M_B=0 \qquad M_B=\underline{-149\ \text{kNm}}$$

$$R_A=20\cdot 4-\frac{149}{8}=\underline{61{,}4\ \text{kN}} \qquad M_{AB}^{max}=\frac{61.4^2}{40}=94$$

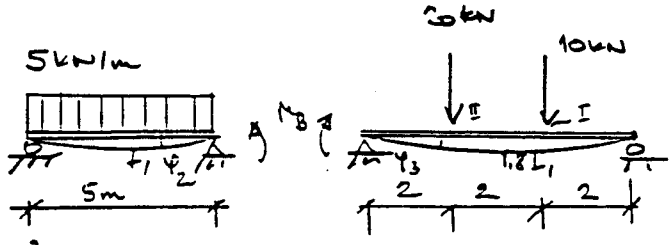
$$R_{BV}=160-61.4=\underline{98{,}6\ \text{kN}}$$

$$R_{BH}=20\cdot 3+\frac{149}{6}=\underline{114.8\ \text{kN}} \qquad M_{BC}^{max}=\frac{65.2^2}{60}=71$$

$$R_{BV}=180-114.8=\underline{65{,}2\ \text{kN}} \qquad M^0_{AB}=\frac{20\cdot 8^2}{8}=\underline{160\ \text{kNm}}$$

$$M^0_{BC}=\frac{30\cdot 6^2}{8}=\underline{135\ \text{kNm}}$$

Ex 186



$\varphi_2 = -\varphi_3$

$$\frac{5}{6EI_1}\left(\frac{5\cdot 5^2}{4} + 2M_B\right) + \frac{6}{6\cdot 1{,}8EI_1}\left(\frac{20\cdot 2\cdot 4\cdot 10}{36} + \frac{10\cdot 4\cdot 2\cdot 8}{36} + 2M_B\right) = 0$$

$$156{,}25 + 10M_B + 207{,}41 + 6{,}67M_B = 0$$

$$\underline{M_B = -21{,}82 \text{ kNm}}$$

$$R_A = \frac{25}{2} - \frac{21.8}{5} = 8{,}14 \text{ kN}$$

$$R_{BV} = \qquad = 16.86$$

$$M_I = 9{,}7\cdot 2 = \underline{19{,}4 \text{ kNm}}$$

$$M_{II} = 9{,}7\cdot 4 - 10\cdot 2 = \underline{18{,}8 \text{ kNm}}$$

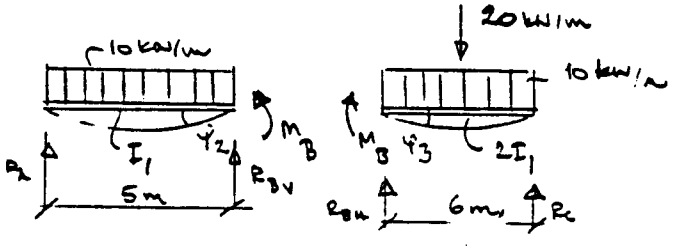
$$R_{BH} = \frac{2\cdot 20}{3} + \frac{10}{3} + \frac{21.8}{6} = 20{,}3 \text{ kN}$$

$$R_C \quad \frac{20}{3} + \frac{2\cdot 10}{3} - \frac{21.8}{6} = 9{,}7 \text{ kN}$$

$$M_{AB}^{max} = \frac{8.1^2}{10} = \underline{6{,}6 \text{ kNm}}$$

$$M_{AB}^{o} = \frac{5\cdot 5^2}{8} = \underline{15.6 \text{ kNm}}$$

Ex 187



$\varphi_2 = -\varphi_3$

$$\frac{5}{6EI_1}\left(\frac{10\cdot 5^2}{4} + 2M_B\right) + \frac{6\cdot 3}{12EI_1}\left(\frac{10\cdot 6^2}{4} + \frac{3\cdot 20\cdot 6}{8} + 2M_B\right) = 0$$

$$312{,}5 + 10M_B + 405 + 6M_B = 0 \qquad \underline{M_B = -44{,}8 \text{ kNm}}$$

$$R_A = \frac{50}{2} - \frac{44.8}{5} = \underline{16{,}0 \text{ kN}}$$

$$M_{BC}^{max} = 32{,}5\cdot 3 - 40\cdot 3\cdot 1{,}5 = \underline{52{,}5 \text{ kN}}$$

$$R_{BV} = \frac{50}{2} + \frac{44.8}{5} = \underline{34{,}0 \text{ kN}}$$

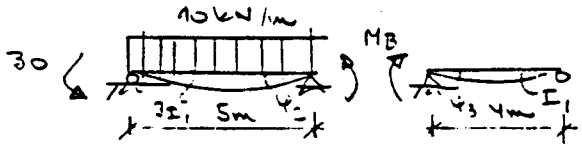
$$M_{AB}^{o} = \frac{10\cdot 25}{8} = 31{,}3$$

$$R_C = 10 + 30 - \frac{44{,}8}{6} = \underline{32{,}5 \text{ kN}}$$

$$M_{BI}^{o} = M_{IC}^{o} = \frac{40\cdot 3^2}{8} = 11{,}3$$

$$R_{BH} = 10 + 30 + \frac{44{,}8}{6} = \underline{47{,}5 \text{ kN}}$$

$$M_{AB}^{max} = \frac{16^2}{20} = \underline{12{,}8 \text{ kNm}}$$



$$\frac{5}{6\cdot 3EI_1}\left(\frac{10\cdot 5^2}{4} - 30 + 2M_B\right) + \frac{4\cdot 3}{6EI_1}\cdot 2M_B = 0$$

$$312.5 - 150 + 10M_B + 24M_B = 0 \qquad \underline{M_B = 4{,}78 \text{ kNm}}$$

$$R_A = \frac{50}{2} + \frac{30-4.8}{5} = \underline{30.0 \text{ kN}}$$

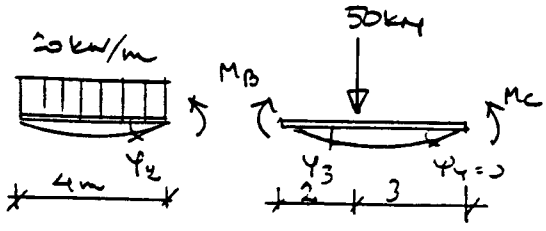
$$R_{BV} = 25 + \frac{4.8-30}{5} = \underline{20.0 \text{ kN}}$$

$$R_{BH} = \frac{4.8}{4} = \underline{1.2 \text{ kN}}$$

$$R_C = -\underline{1.2 \text{ kN}}$$

$$M^{0}_{MB} = \frac{10\cdot 5^2}{8} = \underline{31{,}3 \text{ kNm}}$$

$$M^{max}_{AB} = \frac{20^2}{20} - 4.8 = \underline{15.2 \text{ kNm}}$$



$\varphi_2 = -\varphi_3 \qquad \varphi_4 = 0$

$$\frac{4}{4EI}\left(\frac{20\cdot 4^2}{4} + 2M_B\right) + \frac{5}{6EI}\left(\frac{50\cdot 2.3(8)}{25} + 2M_B + M_C\right) = 0$$

$$\frac{5}{6EI}\left(\frac{50\cdot 3\cdot 2.7}{25} + 2M_C + M_B\right) = 0$$

$$\begin{cases} 320 + 8M_B + 480 + 10M_B + 5M_C = 0 \\ -84 - 2M_C = M_B \end{cases}$$

$$800 + 5M_C + 18(-84 - 2M_C) = 0$$

$$-712 - 31M_C = 0 \qquad \underline{M_C = -23.0 \text{ kNm}}$$

$$M_B = +46 - 84 = \underline{-38{,}1 \text{ kNm}}$$

$$R_1 = 40 - \frac{38.1}{4} = \underline{30{,}5}$$

$$R_2 = 40 + \quad = \underline{49.5}$$

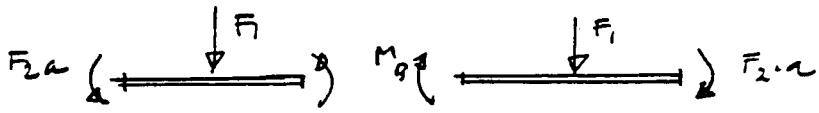
$$R_3 = \frac{3}{5}50 + \frac{38{,}1-23}{5} = \underline{33.0}$$

$$R_4 = \frac{2}{5}50 - \quad = \underline{17.0}$$

$$M^{max}_{AB} = \frac{30.5^2}{40} = \underline{23{,}2 \text{ kNm}}$$

$$M^{max}_{BC} = 17{,}0\cdot 3 - 23.0 = \underline{28.0 \text{ kNm}}$$

Ex 190

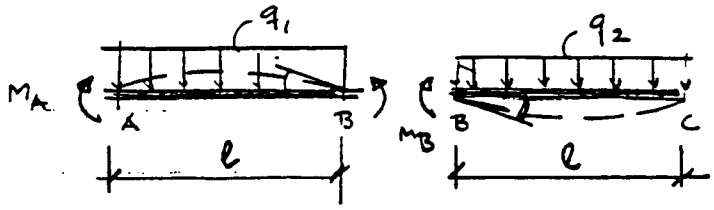


$$\theta_B = \frac{2a}{6EI}\left(\frac{3F_1 \cdot 2a}{8} - F_2 a + 2M_B\right) = 0 \qquad M_B = -F_2 a$$

$$\frac{3F_1 \cdot 2\cancel{a}}{8} - F_2 \cdot \cancel{a} - 2F_2 \cdot \cancel{a} = 0$$

$$\frac{\cancel{3}F_1}{4} = \cancel{3}F_2 \qquad \underline{F_1 = 4F_2}$$

Ex 191



$$\theta_{BH} = \frac{l}{6EI}\left(\frac{q_2 \cdot l^2}{4} + 2M_B\right) = 0 \qquad M_B = -\frac{q_2 l^2}{8}$$

$$\theta_{BV} = \frac{l}{6EI}\left(\frac{q_1 l^2}{4} + 2M_B + M_A\right) = 0 \qquad M_A = -2M_B - \frac{q_1 l^2}{4} =$$

$$= +\frac{q_2 l^2}{4} - \frac{q_1 l^2}{4}$$

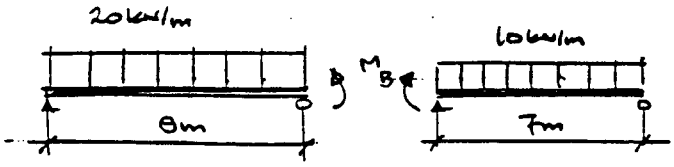
$$\theta_A = \frac{l}{6EI}\left(\frac{q_1 l^2}{4} + 2M_A + M_B\right) = 0$$

$$\frac{q_1 l^2}{4} + \frac{q_2 l^2}{2} - \frac{q_1 l^2}{2} - \frac{q_2 l^2}{8}$$

$$2q_1 + 4q_2 = 4q_1 + q_2$$

$$3q_2 = 2q_1 \qquad \underline{q_1 = 1.5\, q_2}$$

Ex 192



$$\frac{8}{6EI}\left(\frac{20\cdot 8^2}{4}+2M_B\right)=\frac{7}{6EI}\left(-\frac{10\cdot 7^2}{4}-2M_B\right)$$

$$2560+16M_B=-857{,}5-14M_B$$

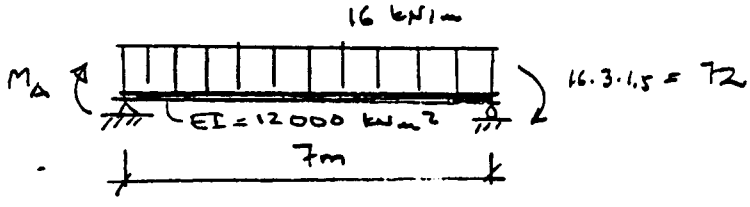
$$30M_B=-3417{,}5$$

$$\underline{M_B=-113{,}9\ \text{kNm}}$$

$$y_{mid}=\frac{5\cdot 20\cdot 8^4}{384EI}-\frac{113{,}9\cdot 8^2}{16\cdot EI}=\frac{1066{,}7-455{,}6}{EI}=\frac{611}{EI}$$

$$y_{mid}=\frac{611}{16000}=\underline{38{,}2\ \text{mm}}$$

Ex 193



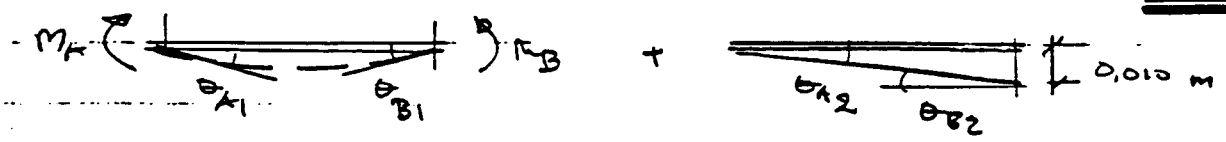
$$\frac{7}{6EI}\left(\frac{16\cdot 7^2}{4}+2M_A-72\right)=0 \qquad \underline{M_A=-62\ \text{kNm}}$$

$$\underline{M_B=-72\ \text{kNm}}$$

$$y_{mid}=\frac{5\cdot 16\cdot 7^4}{384EI}-\frac{(62+72)\cdot 7^2}{16EI}=\frac{500{,}2-410{,}4}{EI}=$$

$$=\frac{89{,}8}{12000}=0{,}0075=\underline{7{,}5\ \text{mm}}$$

Ex 194



$$\theta_A = \frac{4}{6EI}\left(2M_A + M_B + \frac{6EI \cdot 0{,}01}{4^2}\right) = 0$$

$$\theta_B = \frac{4}{6EI}\left(2M_B + M_A - \frac{6EI \cdot 0{,}01}{4^2}\right) = 0$$

$$M_B = -2M_A - 0{,}00375\ EI$$

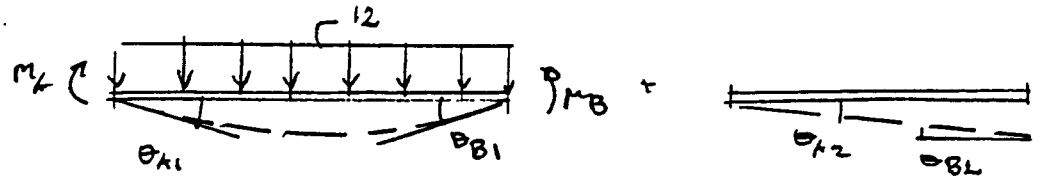
$$-4M_A - 0{,}00375\,EI + M_A - 0{,}00375\,EI = 0$$

$$M_A = -0{,}00375 \cdot EI$$

$EI = 210 \cdot 182{,}6 = 38346$ $\quad M_A = -0{,}00375 \cdot 38346 = \underline{-144\ kNm}$

$M_B = \underline{+144\ kNm}$

Ex 195



$$\theta_A = \frac{l}{6EI}\left(\frac{12 \cdot 5^2}{4} + 2M_A + M_B + \frac{6EI \cdot 0{,}01}{5^2}\right) = 0$$

$$\theta_B = \frac{l}{6EI}\left(\frac{12 \cdot 5^2}{4} + 2M_B + M_A - \frac{6EI \cdot 0{,}01}{5^2}\right) = 0$$

$$\begin{cases} 75 + 2M_A + M_B + 0{,}0024\ EI = 0 \\ 75 + 2M_B + M_A - 0{,}0024\ EI = 0 \end{cases}$$

$$M_B = -75 - 2M_A - 0{,}0024\ EI$$

$$75 - 150 - 4M_A - 0{,}0048\ EI + M_A - 0{,}0024\ EI = 0$$

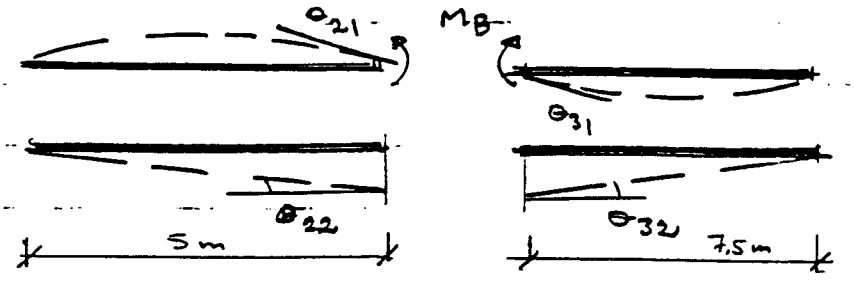
$$-75 - 3M_A - 0{,}0072\ EI = 0$$

$$M_A = -25 - 0{,}0024\ EI$$

$EI = 210 \cdot 1627 = 34167$ $\quad M_A = -25 - 0{,}0024 \cdot 34167 = \underline{-107\ kNm}$

$M_B = -75 + 214 - 0{,}0024 \cdot 34167 = \underline{+57\ kNm}$

Ex 196



$$\theta_{21} - \theta_{22} = \theta_{31} - \theta_{32}$$

$$\theta_{21} = -\frac{5}{6EI} \cdot 2M_B \qquad \theta_{22} = \frac{0{,}015}{5} = 0{,}003$$

$$\theta_{31} = +\frac{7{,}5}{6EI} \cdot 2M_B \qquad \theta_{32} = -\frac{0{,}015}{7{,}5} = -0{,}002$$

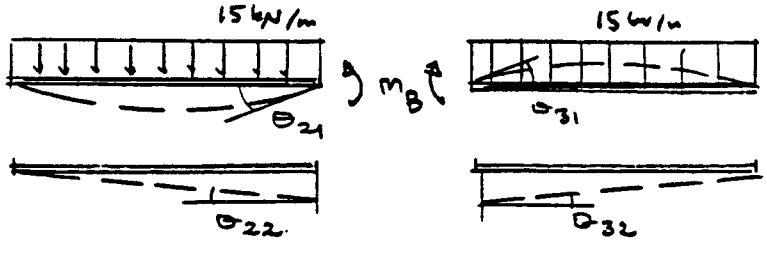
$$-\frac{10 M_B}{6EI} + 0{,}003 = \frac{15 M_B}{6EI} - 0{,}002$$

$$0{,}005 \cdot 6EI = 25 M_B$$

$$M_B = 0{,}0012\, EI =$$

$$EI = 210 \cdot 231{,}3 = 48570 \qquad = 0{,}0012 \cdot 48570 = \underline{58{,}3 \text{ kN}}$$

Ex 197



$$EI = 210 \cdot 136{,}7 = 28707$$

$$\theta_{21} - \theta_{22} = \theta_{31} + \theta_{32}$$

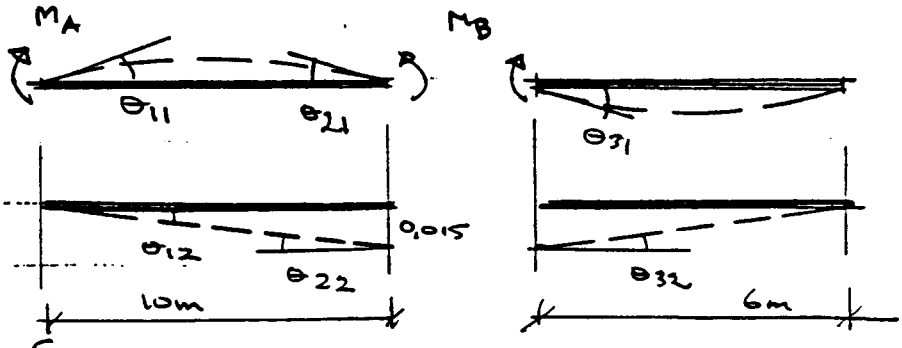
$$\theta_{21} = \frac{8}{6EI}\left(2M_B + \frac{15 \cdot 8^2}{4}\right) \qquad \theta_{22} = \frac{0{,}015}{8}$$

$$\theta_{31} = \frac{8}{6EI}\left(-2M_B - \frac{15 \cdot 8^2}{4}\right) \qquad \theta_{32} = \frac{0{,}015}{8}$$

$$2M_B + 240 - \frac{0{,}015 \cdot 6EI}{8^2} = -2M_B - 240 + \frac{0{,}015 \cdot 6EI}{8^2}$$

$$M_B = -120 + \frac{0{,}015 \cdot 6EI}{64 \cdot 2} = -120 + 0{,}000703\, EI$$

$$= -120 + 20{,}2 = \underline{-99{,}8 \text{ kNm}}$$



$$\begin{cases} \theta_{11} - \theta_{12} = 0 \\ \theta_{21} + \theta_{22} = \theta_{31} - \theta_{32} \end{cases}$$

$EI = 210 \cdot 330{,}9 = 69500$

$$\theta_{11} = \frac{10}{6EI}\left(-2M_A - M_B\right) \qquad \theta_{12} = \frac{0{,}015}{10}$$

$$\theta_{21} = \frac{10}{6EI}\left(-2M_B - M_A\right) \qquad \theta_{22} = \frac{0{,}015}{10}$$

$$\theta_{31} = \frac{6}{6EI}\left(2M_B\right) \qquad \theta_{32} = \frac{0{,}015}{6}$$

$$\begin{cases} \frac{10}{6EI}\left(-2M_A - M_B\right) - \frac{0{,}015}{10} = 0 \\ \frac{10}{6EI}\left(-2M_B - M_A\right) + \frac{0{,}015}{10} = \frac{6 \cdot 2M_B}{6EI} - \frac{0{,}015}{6} \end{cases}$$

$$2M_A + M_B + \frac{0{,}015 \cdot 6EI}{100} = 0 \qquad \boxed{M_B = -2M_A - 0{,}0009\,EI}$$

$$= -2M_A - 62{,}5$$

$$-2M_B - M_A + \frac{0{,}015 \cdot 6EI}{100} = \frac{12M_B \cancel{6EI}}{\cancel{6EI} \cdot 10} - \frac{0{,}015 \cdot \cancel{6}EI}{\cancel{6} \cdot 10}$$

$$-3{,}2M_B - M_A = -0{,}0009\,EI - 0{,}0015\,EI = -0{,}0024\,EI$$

$$\boxed{3{,}2M_B + M_A = 166{,}8}$$

$$3{,}2\left(-2M_A - 62{,}5\right) + M_A = 166{,}8$$

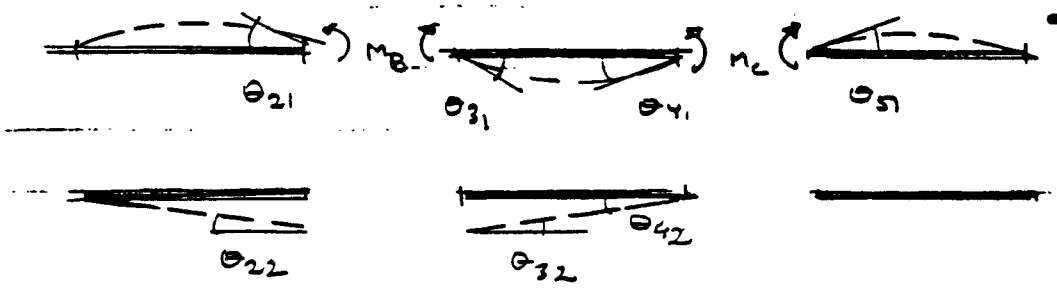
$$-6{,}4\,M_A - 200 + M_A = 166{,}8$$

$$-5{,}4\,M_A = 366{,}8$$

$$\underline{M_A = -67{,}9\ \text{kNm}}$$

$$M_B = +2 \cdot 67{,}9 - 62{,}5 = \underline{73{,}3\ \text{k}}$$

Ex 199



$$\begin{cases} \theta_{21} + \theta_{22} = \theta_{31} - \theta_{32} \\ \theta_{41} + \theta_{42} = \theta_{51} \end{cases} \qquad EI = 210 \cdot 330{,}9 = 69489$$

$$\theta_{21} = \frac{5}{6EI}(-2M_B) \qquad \theta_{22} = \frac{0{,}015}{5}$$

$$\theta_{31} = \frac{6}{6EI}(2M_B + M_C) \qquad \theta_{32} = \frac{0{,}015}{6}$$

$$\theta_{41} = \frac{6}{6EI}(2M_C + M_B) \qquad \theta_{42} = \frac{0{,}015}{6} \qquad \theta_{51} = \frac{5}{6EI}(-2M_C)$$

$$\begin{cases} -\frac{10M_B}{6EI} + \frac{0{,}015}{5} = \frac{6}{6EI}(2M_B + M_C) - \frac{0{,}015}{6} \quad \Big) \cdot \frac{6EI}{10} \\ \frac{6}{6EI}(2M_C + M_B) + \frac{0{,}015}{6} = -\frac{10M_C}{6EI} \quad \Big) \cdot \frac{6EI}{10} \end{cases}$$

$$\begin{cases} -M_B + 0{,}0018\,EI = 1{,}2M_B + 0{,}6M_C - 0{,}0015\,EI \\ 1{,}2M_C + 0{,}6M_B + 0{,}0015\,EI = -M_C \end{cases}$$

$$\begin{cases} 2{,}2M_B + 0{,}6M_C = 0{,}0033\,EI = 229{,}3 \qquad R \\ 2{,}2M_C + 0{,}6M_B = -0{,}0015\,EI = -104{,}2 \end{cases}$$

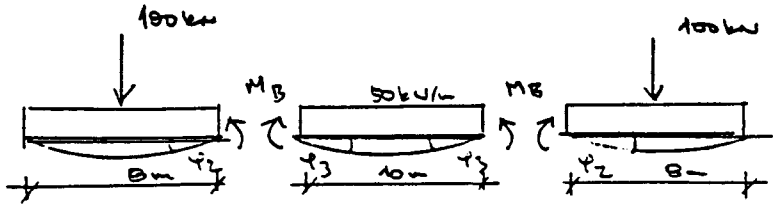
$$\underline{M_C = 382{,}7 - 3{,}667\,M_B}$$

$$841{,}9 - 8{,}067\,M_B + 0{,}6M_B = -104{,}2$$

$$-7{,}467\,M_B = -946 \qquad \underline{M_B = +126{,}7}$$

$$M_C = 382{,}7 - 3{,}667 \cdot 126{,}7 = \underline{-81{,}9 \text{ kNm}}$$

Ex 200



$$\frac{0{,}8}{6EI}\left(\frac{50\cdot 8^2}{4}+2M_B+\frac{3\cdot 100\cdot 8}{8}\right)+\frac{10}{6EI}\left(\frac{50\cdot 10^2}{4}+3M_B\right)=0$$

$640+1.6M_B+240+1250+3M_B=0$ $\qquad$ $M_B=-463$ kNm

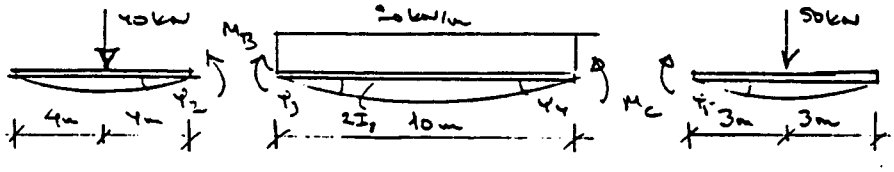
$R_1 = 50+50\cdot 4-463/8 = 192$ kN

$R_2 = \qquad +463/8 = 308$ kN

$R_3 = \qquad = 250$ kN

$M_{AB}^{max} = \frac{192^2}{100} = 369$ kNm $\qquad$ $M_{BC}^{max} = \frac{50\cdot 100}{8}-463 = 162$ kNm

Ex 201



$\varphi_2=-\varphi_3$

$\varphi_4=-\varphi_5$

$$\frac{8}{6EI_1}\left(\frac{3\cdot 40\cdot 8}{8}+2M_B\right)+\frac{10\cdot 5}{6E\cdot 5I_1}\left(\frac{20\cdot 10^2}{4}+2M_B+M_C\right)=0$$

$$\frac{10\cdot 5}{6E\cdot 5I_1}\left(\frac{20\cdot 10^2}{4}+2M_C+M_B\right)+\frac{6}{6EI_1}\left(\frac{50\cdot 3\cdot 6}{8}+2M_C\right)=0$$

$$\begin{cases} 960+16M_B+2500+10M_B+5M_C=0 & 3460+26M_B+5M_C=0 \\ 2500+10M_C+5M_B+675+12M_C=0 & 3175+5M_B+22M_C=0 \end{cases}$$

$$\boxed{M_B=-4.4M_C-635}$$

$3460-26(-4.4M_C-635)-5M_C=0$

$-13050-109.4M_C=0$ $\qquad$ $M_C=-119.3$ kNm $\qquad$ $M_B=-110{,}1$ kNm

$R_1 = 20-\frac{110}{8} = 6{,}2$ kN

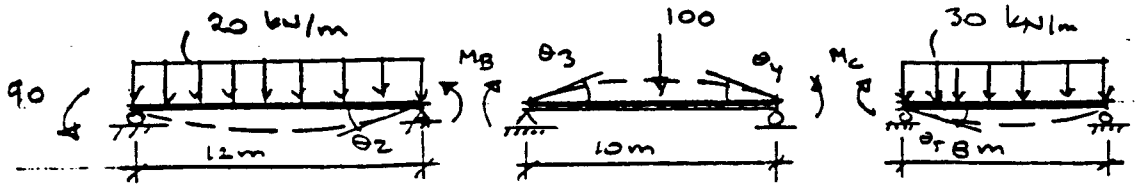
$R_2 = 20+ \qquad = 33.8$ kN

$R_3 = 20\cdot 5+\frac{110.1-119.3}{10} = 99.1$ kN $\Big\}$ $132{,}9$ kN

$R_4 = \qquad = 100.9$ kN $\Big\}$ $145{,}8$ kN

$R_5 = 25+\frac{119.3}{6} = 44.9$

$R_6 = 25- \qquad = 5{,}1$ kN



Ex 202

$\theta_2 = \theta_3 \qquad \theta_4 = \theta_5$

$$\begin{cases} \dfrac{12}{\cancel{6EI}}\left(\dfrac{20\cdot 12^2}{4} + 2M_B - 90\right) = \dfrac{10}{\cancel{6EI}}\left(-\dfrac{3\cdot 100\cdot 10}{8} - 2M_B - M_C\right) \\[2ex] \dfrac{10}{\cancel{6EI}}\left(-\dfrac{3\cdot 100\cdot 10}{8} - 2M_C - M_B\right) = \dfrac{8}{\cancel{6EI}}\left(\dfrac{30\cdot 8^2}{4} + 2M_C\right) \end{cases}$$

$$\begin{cases} 756 + 2{,}4 M_B = -375 - 2M_B - M_C \\ -375 - 2M_C - M_B = 384 + 1{,}6 M_C \end{cases}$$

$$\begin{cases} 1131 + 4.4 M_B + M_C = 0 \\ 759 + 3{,}6 M_C + M_B = 0 \qquad M_B = -759 - 3{,}6 M_C \end{cases}$$

$$1131 - 3339{,}6 - 15{,}84 M_C + M_C = 0 \qquad \underline{M_C = -148{,}8 \text{ kNm}}$$

$$M_B = -759 + 3{,}6\cdot 148{,}8 = \underline{-223{,}2 \text{ kNm}}$$

$$R_A = 20\cdot 3 + \frac{20\cdot 12}{2} + \frac{90 - 223{,}2}{12} = 60 + 108{,}9 = \underline{168{,}9 \text{ kN}}$$

$$R_B = \frac{20\cdot 12}{2} + \frac{223{,}2 - 90}{12} + 50 + \frac{223{,}2 - 148{,}8}{10} = 131{,}1 + 57{,}4 = \underline{188{,}5 \text{ kN}}$$

$$R_C = 50 + \frac{148{,}8 - 223{,}2}{10} + \frac{30\cdot 8}{2} + \frac{148{,}8}{8} = 42{,}6 + 138{,}6 = \underline{181{,}2 \text{ kN}}$$

640

$$R_D = \frac{30\cdot 8}{2} - \frac{148{,}8}{8} = \underline{101{,}4 \text{ kN}}$$

$$M_{AB}^{max} = \frac{108{,}9^2}{40} - 90 = \underline{206 \text{ kNm}}$$

$$M_{BC}^{max} = 57{,}4\cdot 5 - 223{,}2 = \underline{64 \text{ kNm}}$$

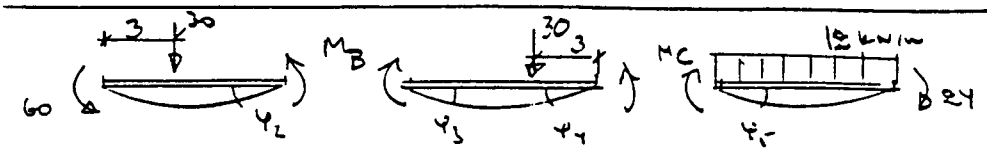
$$M_{CD}^{max} = \frac{101{,}4^2}{60} = \underline{171 \text{ kNm}}$$

forts →

$$M_{AB}^{max} = 6.2 \cdot 4 = \underline{25\,kNm}$$

$$M_{BC}^{max} = \frac{99.1^2}{40} - 110.1 = \underline{135\,kNm}$$

$$M_{CD}^{max} = 5.1 \cdot 3 = \underline{15\,kNm}$$



Ex 203

$$\frac{9}{6EI}\left(\frac{30 \cdot 3 \cdot 6 \cdot 12}{81} + 2M_B - 60\right) + \frac{9}{6EI}\left(\frac{30 \cdot 6 \cdot 3 \cdot 12}{81} + 2M_B + M_C\right) = 0$$

$$\frac{9}{6EI}\left(\frac{30 \cdot 3 \cdot 6 \cdot 15}{81} + 2M_C + M_B\right) + \frac{9}{6EI}\left(\frac{12 \cdot 9^2}{4} + 2M_C - 24\right)$$

$$\begin{cases} 80 + 2M_B - 60 + 80 + 2M_B + M_C = 0 \\ 100 + 2M_C + M_B + 243 + 2M_C - 24 = 0 \end{cases}$$

$$R_0 = \underline{30\,kN}$$

$$\begin{cases} 100 + 4M_B + M_C = 0 \\ 319 + M_B + 4M_C = 0 \end{cases}$$

$$\underline{M_C = -100 - 4M_B}$$

$$319 + M_B - 400 - 16M_B = 0$$

$$\underline{M_B = -5{,}4} \qquad \underline{M_C = -78.4}$$

$$M_{AB} = 26{,}1 \cdot 3 - 60 = \underline{18{,}3}$$

$$M_{BC} = 28{,}1 \cdot 3 - 78.4 = \underline{5{,}9}$$

$$M_{CD} = \frac{48{,}0^2}{24} - 24 = \underline{72}$$

$$R_1 = 20 + \frac{60 - 5{,}4}{9} = 26{,}1\,kN$$

$$R_2 = 10 + \frac{5.4 - 60}{9} = 3{,}9\,kN$$

$$R_3 = 10 + \frac{5{,}4 - 78{,}4}{9} = 1{,}9\,kN$$

$$R_4 = 20 + \frac{78{,}4 - 5{,}4}{9} = 28{,}1\,kN$$

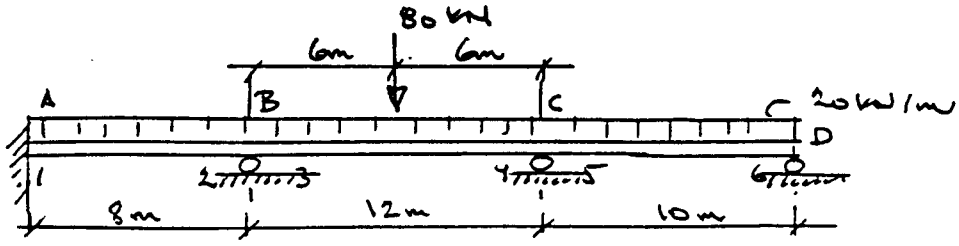
$$R_5 = 12 \cdot 4{,}5 + \frac{78{,}4 - 24}{9} = 60{,}0$$

$$R_6 = 12 \cdot 4{,}5 + \frac{24 - 78{,}4}{9} = 48{,}0$$

$$R_7 = \underline{24\,kN}$$

$$\begin{cases} R_A = \underline{56.1\,kN} \\ R_B = \underline{5{,}8\,kN} \\ R_C = \underline{88.1\,kN} \\ R_D = \underline{72{,}0\,kN} \end{cases}$$

Ex 204



$r_1 = \dots \quad r_5 = -0.5 \qquad s_6 = 0$

$s_2 = \frac{4EI}{8} = 0.5\,EI \qquad s_3 = s_4 = \frac{4EI}{12} = 0.333\,EI \qquad s_5 = \frac{3EI}{10} = 0.3\,EI$

$f_2 = \frac{0.5}{0.8333} = 0.60 \qquad f_3 = 0.40$

$f_4 = \frac{0.3333}{0.6333} = 0.526 \qquad f_5 = 0.474$

$M_1^s = M_2^s = -\frac{20 \cdot 8^2}{12} = \underline{-106.7 \text{ kNm}}$

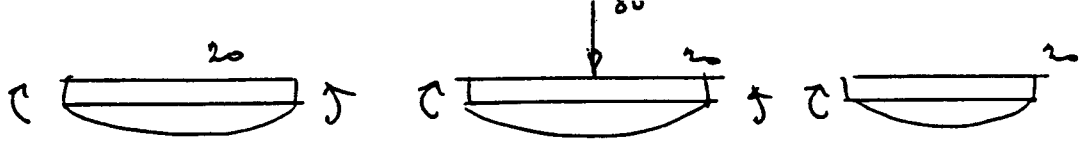
$M_3 = M_4 = -\frac{20 \cdot 12^2}{12} - \frac{80 \cdot 12}{8} = -240 - 120 = \underline{-360 \text{ kNm}}$

$M_5 = \frac{20 \cdot 10^2}{8} = \underline{-250 \text{ kNm}}$

		0,60	0.40		0,526	0.474	
-106,7	-106,7		-360		-360	-250	0
+76,0 ←	-152,0		+101.3	→	-50,7		
			-42,3		+84,5	-76.2	0
+12,7 ←	-25.4		+16.9		-8,5		
			-2.2		+4.5	-4.0	0
+0,7 ←	-1.3		+0.9		-0.4		
			-0.1		+0.2	-0.2	
0,0	-0.1						
-17,3	-285,5		-285.5		-330,4	-330,4	0

Svar: $M_A = -17$ kNm $M_B = -285$ kNm $M_C = -330$ kNm

forts →



$$\frac{8}{\cancel{EI}}\left(2M_A + M_B + \frac{20\cdot 8^2}{4}\right) = 0$$

$$\frac{\cancel{8}}{\cancel{EI}}\left(2M_B + M_A + \frac{20\cdot 8^2}{4}\right) + \frac{\overset{1,5}{\cancel{12}}}{\cancel{EI}}\left(2M_B + M_C + \frac{3\cdot 80\cdot 12}{8} + \frac{20\cdot 144}{4}\right) = 0$$

$$\frac{12}{\cancel{EI}}\left(2M_C + M_B + \frac{3\cdot 80\cdot 12}{8} + \frac{20\cdot 144}{4}\right) + \frac{\cancel{10}}{\cancel{EI}}\left(2M_C + \frac{20\cdot 100}{4}\right) = 0$$

$$\boxed{M_A = -0,5M_B - 160}$$

$$2M_B + M_A + 320 + 3M_B + 1,5M_C + 540 + 1080 = 0$$

$$2,4M_C + 1,2M_B + 432 + 864 + 2M_C + 500 = 0$$

$$2M_B - 0,5M_B - 160 + 1920 + 3M_B + 1,5M_C = 0$$

$$\begin{cases} 4,4M_C + 1,2M_B + 1796 = 0 \\ 4,5M_B + 1,5M_C + 1780 = 0 \end{cases}$$

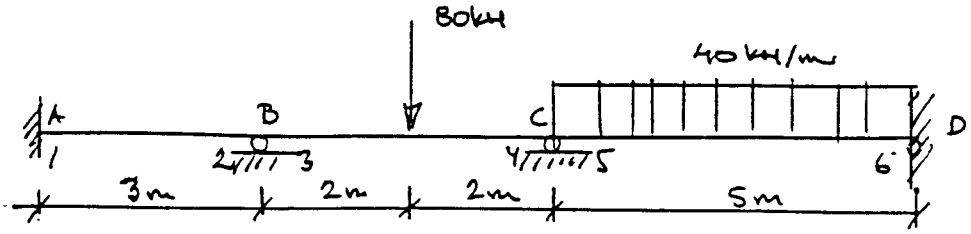
$$\boxed{M_C = -3M_B - 1186,7}$$

$$-13,2M_B - 5221,5 + 1,2M_B + 1796 = 0$$

$$M_B = -\frac{3425,5}{12} = \underline{-285,46}$$

$M_C = + \underline{-330,3 \text{ kNm}}$

$\underline{M_A = -17,3}$



$r_1 = \dots r_6 = -0{,}5$

$S_2 = \frac{4EI}{3} = 1{,}333\ EI \qquad S_3 = S_4 = \frac{4EI}{4} = EI \qquad S_5 = \frac{4EI}{5} = 0{,}8\ EI$

$f_2 = \frac{1{,}333}{2{,}333} = 0{,}571 \qquad f_3 = \frac{1}{2{,}333} = 0{,}429$

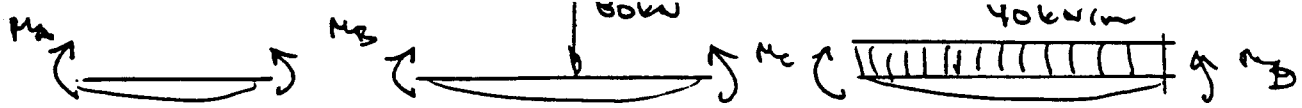
$f_4 = \frac{1}{1{,}8} = 0{,}556 \qquad f_5 = 0{,}444$

$M_1^S = M_2^S = 0 \qquad M_3^S = M_4^S = -\frac{80 \cdot 4}{8} = -40\ kNm$

$M_5^S = M_6^S = -\frac{40 \cdot 25}{12} = -83{,}3\ kNm$

	0.571	0.429	0.556	0.444	
0	0	-40	-40	-83,3	-83,3
		+12.0 ←	-24,0	+19.2	→ -9.6
+8,0 ←	-16,0	+12,0 →			
			-6.0		
		-1.7	+3,3	-2.7	→ +1.3
+0.5 ←	-1.0	+0.7 →	-0.4		
		←	+0.2	-0.2	→ +0.1
+8,5	-17.0	-17,0	-67,0	-67.0	-91,5
		-40	-40	-83,33	-83,33
		+12,05	-24.09	+19.24	→ -9.62
		+11.99 →	-6.00		
		-1,67 ←	+3,34	-2.66	→ +1,33
		+0,72	-0,36		
		-0.10	+0.20	-0.16	→ +0,08
		+0.04	-0.02		
			+0.01	-0.01	
	-17,0	-17.0	-66,9	-66.9	-91.5

forts →



$$\frac{3}{6EI}\left(2M_A + M_B\right) = 0$$

$$\frac{3}{\cancel{6EI}}\left(2M_B + M_A\right) + \frac{4}{\cancel{6EI}}\left(2M_B + M_C + \frac{3\cdot 80\cdot 4}{8}\right) = 0$$

$$\frac{4}{\cancel{6EI}}\left(2M_C + M_B + \frac{3\cdot 80\cdot 4}{8}\right) + \frac{5}{\cancel{6EI}}\left(2M_C + M_D + \frac{40\cdot 25}{4}\right) = 0$$

$$\frac{5}{6EI}\left(2M_D + M_C + \frac{40\cdot 25}{4}\right) = 0$$

$$\boxed{M_A = -0{,}5\,M_B}$$

$$\boxed{M_D = -0{,}5\,M_C - 125}$$

$$0{,}75\left(\overset{1{,}5\,M_B}{2M_B - 0{,}5M_B}\right) + 2M_B + M_C + 120 = 0$$

$$0{,}8\left(2M_C + M_B + 120\right) + 2M_C - 0{,}5M_C - 125 + 250 = 0$$

$$\begin{cases} 3{,}125\,M_B + M_C + 120 = 0 \\ 3{,}1\,M_C + 0{,}8\,M_B + 221 = 0 \end{cases} \qquad M_C = -120 - 3{,}125\,M_B$$

$$-372 - 9{,}69\,M_B + 0{,}8\,M_B + 221 = 0$$

$$M_B = -\frac{8{,}89}{151} = \underline{-16{,}985 \text{ kNm} = -17{,}0}$$

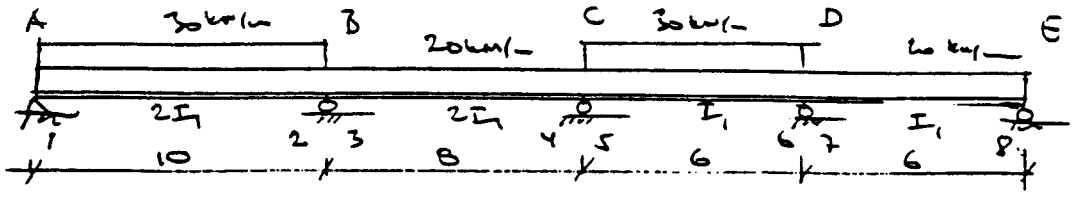
$$M_C = -120 + 3{,}125\cdot 16{,}985 = \underline{-66{,}92 \text{ kNm}} = \underline{-66{,}9 \text{ kNm}}$$

$$M_D = \underline{-91{,}5 \text{ kNm}}$$

$$M_A = \underline{+8{,}5 \text{ kNm}}$$

<u>Svar:</u> $M_A = +8{,}5$ kNm $\quad M_B = -17{,}0$ kNm

$M_C = -66{,}9$ kNm $\quad M_D = -91{,}5$ kNm



$S_2 = \frac{3E \cdot 2I_1}{10} = 0{,}6EI_1$ $\quad S_3 = S_4 = \frac{4E \cdot 2I_1}{8} = EI_1$

$S_5 = S_6 = \frac{4EI_1}{6} = 0.667\,EI_1$ $\quad S_7 = \frac{3EI_1}{6} = 0{,}5\,EI_1$

$f_2 = \frac{0.6}{1.6} = 0.375$ $\quad f_3 = \frac{1}{1.6} = 0.625$ $\quad f_4 = \frac{1}{1.667} = 0.60$

$f_5 = 0.40$ $\quad f_6 = \frac{0.667}{1{,}167} = 0.571$ $\quad f_7 = 0.429$

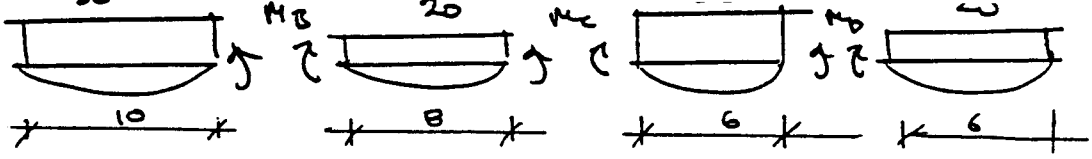
$M_2 = -\frac{30 \cdot 10^2}{8} = -375$ / $M_3 = M_4 = -\frac{20 \cdot 8^2}{12} = -106{,}7$ $\quad M_5 = M_6 = -\frac{20 \cdot 6^2}{12} = -90$

$M_7 = -\frac{20 \cdot 6^2}{8} = -90$

	0.375	0.625		0.60	0.40		0.571	0.429
0	-375	-106,7		-106,7	-90		-90	-90
	+100,6	-167,7		+83.8				
		+20,1		-40,3	+26.8		-13,4	
	+7.5	-12,6	→	+6,3	-3.8		+7.7	-5,7
		+3,0		-6.1	+4.0		-2.0	
	+1.1	-1.9		+0,9	-0.6		+1,1	-0.9
		+0.5		-0.9	+0.6		-0.3	
	-0.2	+0,3		+0.1	-0.1		+0.2	-0.1
				-0.1	+0.1			
	-265,6	-265,6		-63.0	-63.0		-96,7	-96,7

Svar $\quad M_B = -265$ kNm $\quad M_C = -63$ kNm $\quad M_D = -97$ kNm

fortsatt →



$$\frac{10}{1.25}\left(2M_B + \frac{30\cdot 100}{4}\right) + \frac{0.8}{0.25}\left(2M_B + M_C + \frac{20\cdot 64}{4}\right) = 0$$

$$\frac{8}{2.66}\left(2M_C + M_B + \frac{20\cdot 64}{4}\right) + \frac{6}{6}\left(2M_C + M_D + \frac{30\cdot 36}{4}\right) = 0$$

$$\frac{6}{2.25}\left(2M_D + M_C + \frac{30\cdot 36}{4}\right) + \frac{6}{6}\left(2M_D + \frac{30\cdot 36}{4}\right) = 0$$

$$2M_B + 750 + 1.6M_B + 0.8M_C + 256 = 0$$

$$2M_C + M_B + 320 + 3M_C + 1.5M_D + 405 = 0$$

$$2M_D + M_C + 270 + 2M_D + 180 = 0$$

$$\begin{cases} 3.6M_B + 0.8M_C + 1006 = 0 \\ 5M_C + M_B + 1.5M_D + 725 = 0 \\ M_C + 4M_D + 450 = 0 \end{cases}$$

$$M_B = -0.222M_C - 279.44$$

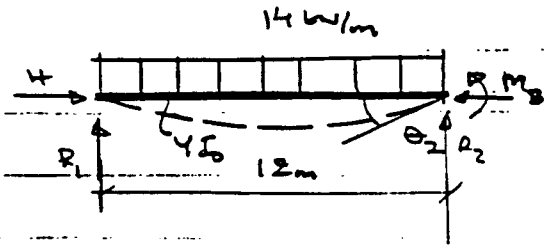
$$\boxed{M_D = -0.25M_C - 112.5}$$

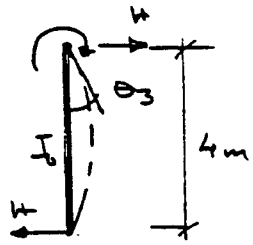
$$5M_C - 279.4 - 0.222M_C - 0.375M_C - 168.8 + 725 = 0$$

$$4.40 \cdot M_C + 276.8 = 0 \qquad M_C = -62.9$$

$$M_D = -16.8 \qquad M_B = -265.4$$

Ex 207





$$\theta_2 = \frac{12}{4\cancel{6EI_0}}\left(\frac{14\cdot 12^2}{4} + 2M_B\right) = \frac{4}{\cancel{6EI_0}}\cdot\left(-2M_B\right) = \theta_3$$

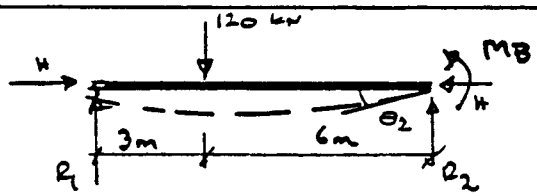
$$1512 + 6M_B = -8M_B \qquad \underline{M_B = -108 \text{ kNm}}$$

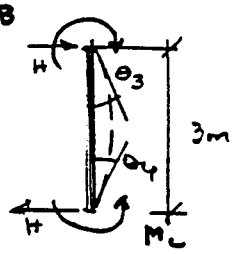
$$R_1 = \frac{14\cdot 12}{2} - \frac{108}{12} = \underline{75 \text{ kN}} \qquad R_2 = 14\cdot 12 - 75 = \underline{93 \text{ kN}}$$

$$\circlearrowleft_3 \quad 108 - H\cdot 4 = 0 \qquad \underline{H = 27 \text{ kN}} \qquad M^0_{12} = \frac{14\cdot 12^2}{8} = \underline{252 \text{ kNm}}$$

$$M^{max}_{12} = \frac{75^2}{28} = \underline{201 \text{ kNm}}$$

Ex 208





$$\theta_2 = \frac{\cancel{9}\,3}{\cancel{6EI}}\left(\frac{120\cdot 3\cdot 6\cdot 12}{81} + 2M_B\right) = \frac{\cancel{3}}{\cancel{6EI}}\left(-2M_B - M_C\right)$$

$$\theta_4 = \frac{3}{6EI}\left(-2M_C - M_B\right) = 0 \qquad \boxed{M_B = -2M_C}$$

$$960 + 6M_B = -2M_B - M_C$$

$$960 = -8M_B - M_C = +16M_C - M_C \qquad \underline{M_C = 64 \text{ kNm}}$$

$$\underline{M_B = -128 \text{ kNm}}$$

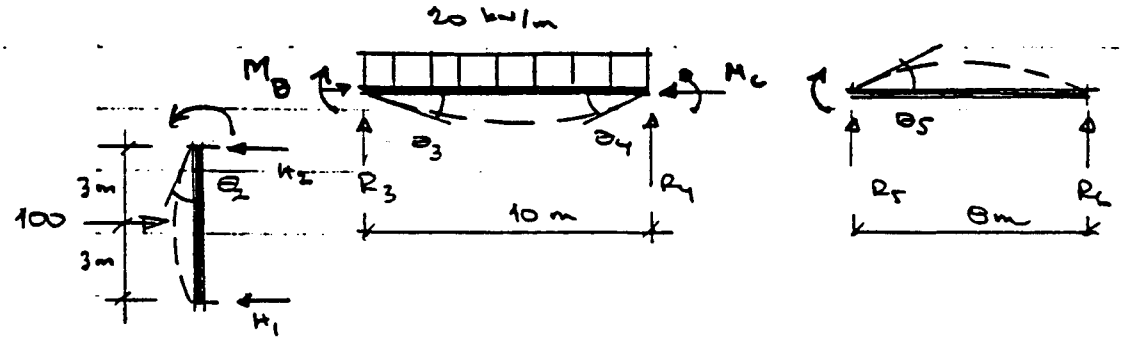
$$R_1 = \frac{120\cdot 2}{3} - \frac{128}{9} = \underline{65{,}8 \text{ kN}}$$

$$R_2 = \frac{120}{3} + \frac{128}{9} = \underline{54{,}2 \text{ kN}} \qquad M^{max}_{12} = 65{,}8\cdot 3 = \underline{197 \text{ kNm}}$$

$$\circlearrowright_4 \quad H\cdot 3 - 128 - 64 = 0$$

$$H = 64 \text{ kN}$$

Ex 209



$$\theta_2 = \frac{6}{6EI}\left(-\frac{3\cdot 100\cdot 6}{8} - 2M_B\right) = \frac{10}{6EI}\left(\frac{20\cdot 10^2}{4} + 2M_B + M_C\right) = \theta_3$$

$$\theta_4 = \frac{10}{6EI}\left(\frac{20\cdot 10^2}{4} + 2M_C + M_B\right) = \frac{8}{6EI}\left(-2M_C\right)$$

$$\begin{cases} -135 - 1.2\,M_B = 500 + 2M_B + M_C \\ 500 + 2M_C + M_B = -1.6M_C \end{cases}$$

$$\begin{cases} 635 + 3.2\,M_B + M_C = 0 \\ 500 + M_B + 3.6M_C = 0 \end{cases} \qquad \underline{M_B = -500 - 3.6\,M_C}$$

$$635 - 1600 - 11.52\,M_C + M_C = 0 \qquad \underline{M_C = -91.7 \text{ kNm}}$$

$$\underline{M_B = -169.8 \text{ kNm}}$$

$$R_3 = \frac{20\cdot 10}{2} + \frac{169.8 - 91.7}{10} = \underline{107.8 \text{ kN}}$$

$$R_4 = 100 - 7.8 = \underline{92.2 \text{ kN}}$$

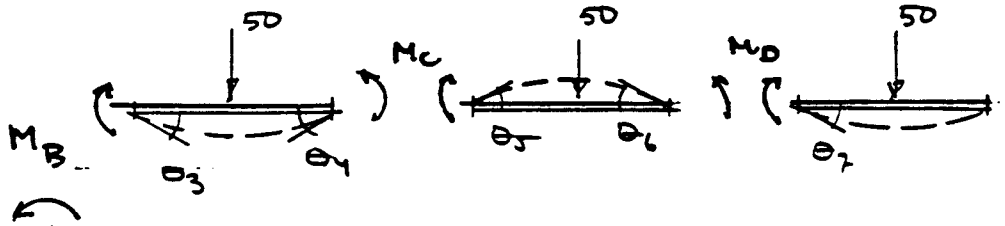
$$R_5 = \frac{91.7}{8} = \underline{11.5 \text{ kN}} \qquad R_6 = -\underline{11.5 \text{ kN}}$$

$$\circlearrowleft \quad 100\cdot 3 + 169.8 - H_2\cdot 6 = 0$$

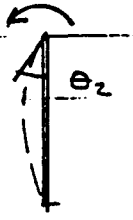
$$H_2 = \underline{78.3 \text{ kN}} \qquad H_1 = \underline{21.7 \text{ kN}}$$

$$M_{12}^{max} = 21.7\cdot 3 = \underline{65 \text{ kNm}} \qquad M_{34}^{max} = \frac{92.2^2}{40} - 91.7 = \underline{121 \text{ kNm}}$$

$$M^0_{34} = \frac{20\cdot 100}{8} = \underline{250 \text{ kNm}}$$



Ex 210



$$\begin{cases} \dfrac{\cancel{3}}{\cancel{6EI}}\left(-8M_B\right) = \dfrac{\cancel{6}}{\cancel{6EI}}\left(2M_B + M_C + \dfrac{3\cdot 50\cdot 6}{8}\right) \\ \dfrac{6}{\cancel{6EI}}\left(2M_C + M_B + \dfrac{3\cdot 50\cdot 6}{8}\right) = \dfrac{8}{\cancel{6EI}}\left(-2M_C - M_D - \dfrac{3\cdot 50\cdot 8}{8}\right) \\ \dfrac{8}{\cancel{6EI}}\left(-2M_D - M_C - \dfrac{3\cdot 50\cdot 8}{8}\right) = \dfrac{6}{\cancel{6EI}}\left(2M_D + \dfrac{3\cdot 50\cdot 6}{8}\right) \end{cases}$$

$$\begin{cases} 0 = 3M_B + M_C + 112{,}5 \\ 1{,}5\cdot M_C + 0{,}75 M_B + 84{,}375 = -2M_C - M_D - 150 \\ -2M_D - M_C - 150 = 1{,}5 M_D + 84{,}375 \end{cases}$$

$$\begin{cases} 0 = 3M_B + M_C + 112{,}5 \\ 3{,}5 M_C + 0{,}75 M_B + M_D + 234{,}375 = 0 \\ M_C + 3{,}5 M_D + 234{,}375 = 0 \end{cases} \qquad \begin{aligned} M_B &= -37{,}5 - \frac{M_C}{3} \\ M_D &= -66{,}96 - 0{,}2857 M_C \end{aligned}$$

$$3{,}5 M_C - 28{,}125 - 0{,}25 M_C - 66{,}96 - 0{,}2857 M_C + 234{,}375 = 0$$

$$2{,}9643 M_C = -139{,}29$$

$$\underline{\underline{M_C = -47{,}0 \text{ kNm}}}$$

$$M_B = -37{,}5 + \frac{47}{3} = \underline{\underline{-21{,}83 \text{ kNm}}} \qquad \underline{\underline{M_D = -53{,}53}}$$

$$H = \frac{21{,}83}{3} = \underline{7{,}3 \text{ kN}} \qquad R_3 = 25 + \frac{21{,}83 - 47{,}0}{6} = \underline{20{,}74} \quad R_4 = \underline{29{,}26}$$

$$R_5 = 25 + \frac{47 - 53{,}53}{8} = \underline{24{,}18} \quad R_6 = \underline{25{,}82}$$

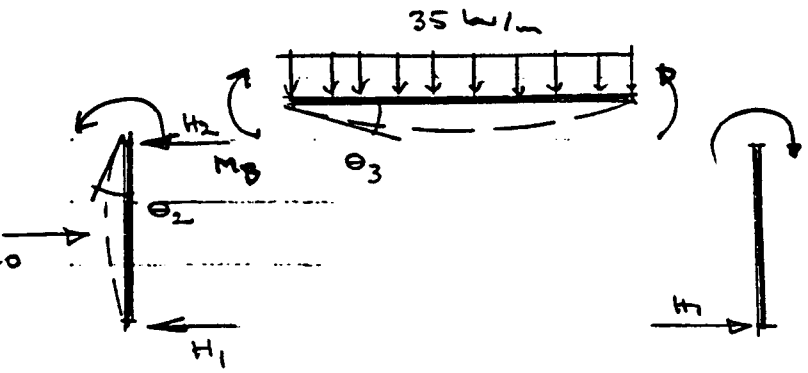
$$R_7 = 25 + \frac{53{,}53}{6} = \underline{33{,}9} \quad R_8 = \underline{16{,}1}$$

$$M_{34} = \underline{\underline{40{,}4}}$$

$$M_{56} = \underline{\underline{49{,}7}}$$

$$M_{78} = \underline{\underline{48{,}2}}$$

Ex 211



$$\theta_2 = \frac{\cancel{4}}{\cancel{6EI}}\left(-2M_B - \frac{3.340.4}{8}\right) = \frac{\cancel{12}^{\,3}}{\cancel{6EI}}\left(\frac{35.12^2}{4} + 2M_B + M_B\right)$$

$$-2M_B - 510 = 3780 + 6M_B + 3M_B$$

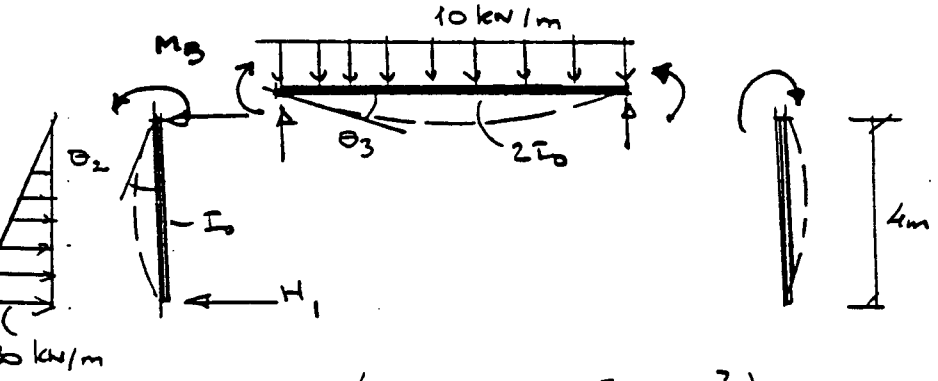
$$4290 = -11M_B$$

$$\underline{M_B = -390 \text{ kNm}}$$

$$R_A = R_B = \frac{35.12}{2} = 210 \text{ kN}$$

$$\circlearrowright 2 \quad 390 + H_1 \cdot 4 - 340 \cdot 2 = 0 \qquad \underline{H_1 = 72{,}5 \text{ kN}}$$

Ex 212



$$\theta_2 = \frac{\cancel{4}}{\cancel{6EI_0}}\left(-2M_B - \frac{7.30.4^2}{60}\right) = \frac{\cancel{8}}{\cancel{6EI_0}}\left(2M_B + M_B + \frac{10.8^2}{4}\right)$$

$$-2M_B - 56 = 3M_B + 160$$

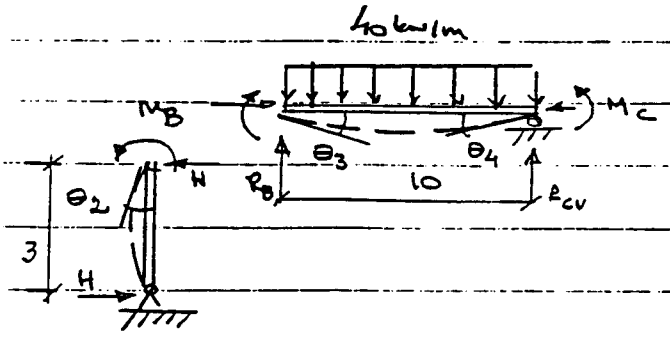
$$5M_B = -216$$

$$\underline{M_B = -43{,}2 \text{ kNm}}$$

$$R_A = R_B = 10.4 = \underline{40 \text{ kN}}$$

$$\circlearrowright 2 \quad 43{,}2 + H_1 \cdot 4 - 30 \cdot \frac{4}{\cancel{2}} \cdot \frac{\cancel{2}.4}{3} = 0 \qquad \underline{H_1 = 29{,}2 \text{ kN}}$$

Ex 213



$\theta_2 = \theta_3$

$\theta_4 = 0$ (SYMMETRY)

$$\theta_2 = \frac{3}{6EI}\left(-2M_B\right) = \frac{10}{6EI}\left(\frac{40 \cdot 10^2}{4} + 2M_B + M_C\right)$$

$$\theta_4 = \frac{10}{6EI}\left(\frac{40 \cdot 10^2}{4} + 2M_C + M_B\right) = 0$$

$$\boxed{M_B = -1000 - 2M_C}$$

$$-0{,}6 M_B = 1000 + 2M_B + M_C$$

$$2{,}6\left(1000 + 2M_C\right) = 1000 + M_C$$

$$1600 = -4{,}2$$

$$\underline{M_C = -381 \text{ kNm}}$$

$$M_B = -1000 + 2 \cdot 381 = \underline{-238 \text{ kNm}}$$

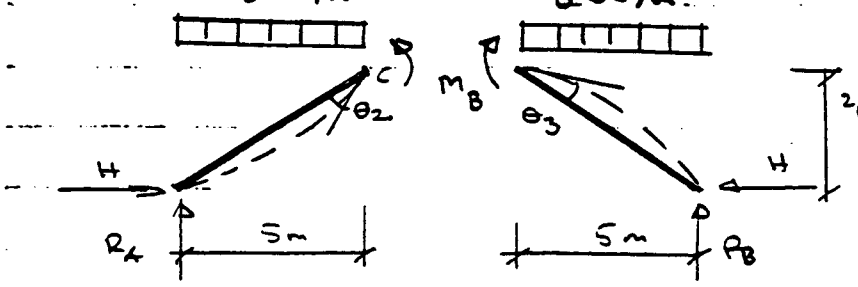
$\circlearrowleft B$ $\quad 238 - H \cdot 3 = 0 \qquad \underline{H = 79 \text{ kN}}$

$\circlearrowleft C$ $\quad R_B \cdot 10 - 238 + 381 - 40 \cdot 10 \cdot 5 = 0$

$$\underline{R_B = 186 \text{ kN}}$$

$\uparrow$ $\quad 186 + R_{CV} - 40 \cdot 10 = 0 \qquad R_{CV} = 214 \text{ kN}$

$$R_C = 2 \cdot 214 = \underline{428 \text{ kN}}$$



Symmetrie yes

$\theta_2 = \theta_3 = 0$

$$\frac{l_s}{6EI}\left(\frac{8\cdot 5^2}{4} + 2M_B\right) = 0 \qquad \underline{M_B = -25 \text{ kNm}}$$

$R_A = R_B = 5\cdot 8 = \underline{40 \text{ kN}}$

$\circlearrowleft C$ $\quad 25 + 40\cdot 5 - H\cdot 2{,}887 - 8\cdot 5\cdot 2{,}5 = 0$

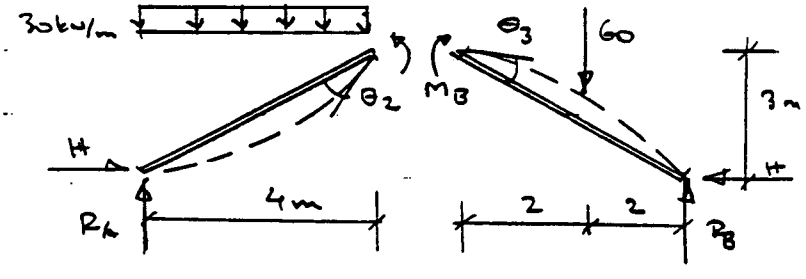
$\underline{H = 43{,}3 \text{ kN}}$

$$\begin{cases} N_A = -43{,}3\cdot\cos 30 - 40\cdot\sin 30 = \underline{-57{,}5 \text{ kN}} \\ V_A = -43{,}3\cdot\sin 30 + 40\cdot\cos 30 = \underline{13{,}0 \text{ kN}} \end{cases}$$

$$\begin{cases} N_C = -43{,}3\cdot\cos 30 = \underline{-37{,}5 \text{ kN}} \\ V_C = -43{,}3\sin 30 = \underline{-21{,}6 \text{ kN}} \end{cases}$$

$M^0_{AC} = 25$

Ex 2.15



$\tan\alpha = \frac{3}{4}$ $\quad \alpha = 36{,}9$

$\sin\alpha = 0{,}6$ $\quad \cos\alpha = 0{,}8$

$$\theta_2 = \frac{l_s}{6EI}\left(\frac{30\cdot 4^2}{4} + 2M_B\right) = \frac{l_s}{6EI}\left(-\frac{3\cdot 60\cdot 4}{8} - 2M_B\right)$$

$120 + 2M_B + 90 + 2M_B = 0$

$\underline{M_B = -52{,}5 \text{ kNm}}$

$\circlearrowright A$ $\quad 30\cdot 4\cdot 2 + 60\cdot 6 - R_B\cdot 8 = 0$ $\qquad \underline{R_B = 75 \text{ kN}}$ $\quad \underline{R_A = 105 \text{ kN}}$

$\circlearrowleft C$ $\quad 52{,}5 + 75\cdot 4 - H\cdot 3 - 60\cdot 2 = 0$ $\qquad \underline{H = 77{,}5 \text{ kN}}$

$N_A = -77{,}5\cdot 0{,}8 - 105\cdot 0{,}6 = \underline{-125 \text{ kN}}$

$V_A = -77{,}5\cdot 0{,}6 + 105\cdot 0{,}8 = \underline{+37{,}5 \text{ kN}}$

$M^0_{AC} = 60$

$N_{CV} = -125 + 30\cdot 4\cdot 0{,}6 = \underline{-53 \text{ kN}}$

$V_{CV} = 37{,}5 - 30\cdot 4\cdot 0{,}8 = \underline{-58{,}5 \text{ kN}}$

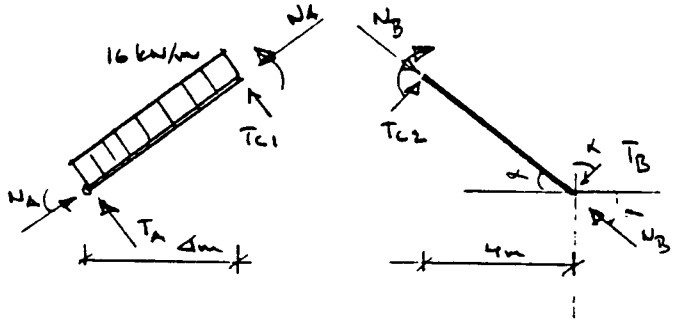
Kollat

$N_B = -77{,}5\cdot 0{,}8 - 75\cdot 0{,}6 = \underline{-107 \text{ kN}}$

$V_B = -75\cdot 0{,}8 + 77{,}5\cdot 0{,}6 = \underline{-13{,}5 \text{ kN}}$

$N_{CH} = -107 + 60\cdot 0{,}6 = \underline{-71 \text{ kN}}$

$V_{CH} = -13{,}5 + 60\cdot 0{,}8 = \underline{+34{,}5 \text{ kN}}$



$\tan\alpha = \frac{3}{4}$ $\quad \sin\alpha = 0.6$

$\cos\alpha = 0.8$

$\alpha = 73.74$ $\quad \sin 2\alpha = 0.96$

$\cos 2\alpha = 0.280$

$$\frac{5}{6EI}\left(\frac{16\cdot 5^2}{4} + 2M_B\right) - \frac{5}{6EI}\cdot 2M_B = 0$$

$$\underline{M_B = -25\ \text{kNm}} \qquad M_0 = \frac{16\cdot 25}{8} = \underline{50\ \text{kNm}}$$

$$T_B = \frac{25}{5} = 5\ \text{kN} \qquad T_A\cdot 5 - 16\cdot 5\cdot 2.5 + 25 = 0 \qquad \underline{T_A = 35\ \text{kN}}$$

$$T_{C1} = 80 - 35 = 45\ \text{kN}$$

Hela ramen

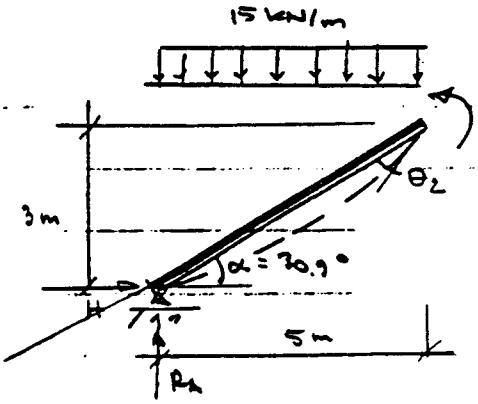
$$\circlearrowleft A \quad \cancel{16}^{2}\cdot 5\cdot \cancel{2}.5 + (T_B\cdot\cos\alpha - N_B\cdot\sin\alpha)\,\cancel{4} = 0$$

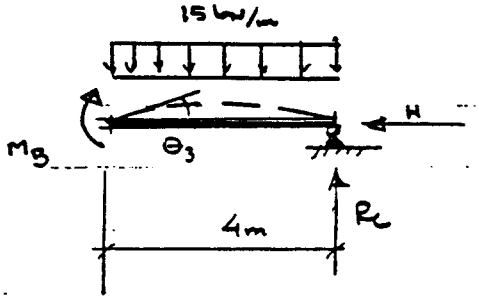
$$25 + 5\cdot 0.8 = N_B\cdot 0.6 \qquad \underline{N_B = 48.3\ \text{kN}}$$

$$\uparrow \quad N_A\cdot \overset{0.6}{\sin\alpha} + 35\cdot 0.8 - 16\cdot 5\cdot 0.8 + 48.3\cdot 0.6 - 5\cdot 0.8 = 0 \qquad \underline{N_A = 18.4\ \text{kN}}$$

$$\left(\longrightarrow \quad N_A\cdot 0.8 + \underset{48}{16\cdot 5\cdot 0.6} - 48.3\cdot 0.8 - 40\cdot 0.6 = 0\right)$$

Ex 2:7





$l_1 = 5,831$ m

$$\frac{l_1}{6EI}\left(\frac{15 \cdot 5^2}{4} + 2M_B\right) = \frac{4}{6EI}\left(-\frac{15 \cdot 4^2}{4} - 2M_B\right)$$

$$\frac{5,831}{4}\left(93,75 + 2M_B\right) + 60 + 2M_B = 0$$

$$136,66 + 2,916 M_B + 60 + 2M_B = 0$$

$$\underline{M_B = -40 \text{ kNm}}$$

HÖGRA DELEN

$$\curvearrowright -40 + 15 \cdot 4 \cdot 2 - R_C \cdot 4 = 0 \qquad \underline{R_C = 20 \text{ kN}} \qquad \underline{R_A = 115 \text{ kN}}$$

VÄNSTRA DELEN

$$\curvearrowright 115 \cdot 5 - 15 \cdot 5 \cdot 2,5 - H \cdot 3 + 40 = 0 \qquad \underline{H = 142,5 \text{ kN}}$$

$$N_A = -142,5 \cdot \cos 30,9 - 115 \cdot \sin 30,9 = -181 \text{ kN}$$

$$V_A = -142,5 \cdot \sin 30,9 + 115 \cdot \cos 30,9 = +25,5 \text{ kN}$$

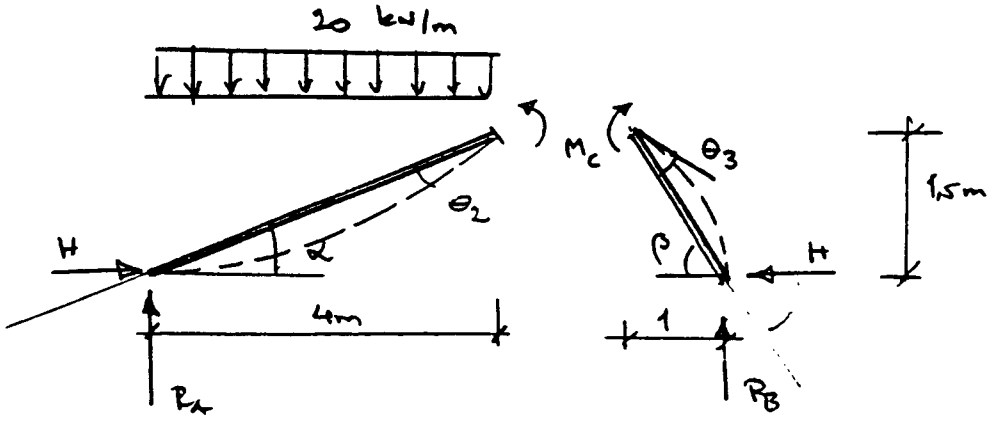
$$N_{CV} = -181 + 15 \cdot 5 \cdot \sin 30,9 = -142 \text{ kN}$$

$$V_{CV} = -15 \cdot 5 \cdot \cos 30,9 + 25,5 = -39 \text{ kN}$$

$$M^0_{AB} = \frac{15 \cdot 5^2}{8} = 47 \text{ kNm} \qquad M^0_{BC} = \frac{15 \cdot 4^2}{8} = 30 \text{ kNm}$$

$$q_s = 15 \cdot \cos 30,9^2 = 11,04 \qquad M^{max}_{AB} = \frac{25,5^2}{2 \cdot 11,04} = 29$$

Ex 21B



$$\tan\alpha = \frac{1,5}{4} \qquad \alpha = 20,56^\circ$$

$$\tan\beta = \frac{1,5}{1} \qquad \beta = 56,31^\circ$$

$$\frac{L_1}{6EI_2}\left(\frac{20\cdot 16}{4} + 2M_C\right) + \frac{L_2}{6EI_1}\cdot 2M_C = 0$$

$$\frac{4,272}{\cancel{6EI_2}\cdot 25}\left(80 + 2M_C\right) + \frac{1,803}{\cancel{6EI_1}}\cdot 2M_C = 0$$

$$\underline{M_C = -19,5 \text{ kNm}}$$

$\curvearrowright A$ $\quad 20\cdot 4\cdot 2 - R_B\cdot 5 = 0 \qquad \underline{R_B = 32 \text{ kN}}$

$\uparrow \quad -20\cdot 4 + R_A + 32 = 0 \qquad \underline{R_A = 48 \text{ kN}}$

$\curvearrowright C$ $\quad -19,5 + H\cdot 1,5 - 32\cdot 1 = 0 \qquad \underline{H = 34,3 \text{ kN}}$

$$N_A = -34,3\cos 20,56^\circ - 48\cdot\sin 20,56 = \underline{-49,0 \text{ kN}}$$

$$V_A = +48\cdot\cos 20,56 - 34,3\cdot\sin 20,56 = \underline{+32,9 \text{ kN}}$$

$$N_{CV} = -49,0 + 20\cdot 4\cdot\sin 20,56 = \underline{-20,9 \text{ kN}}$$

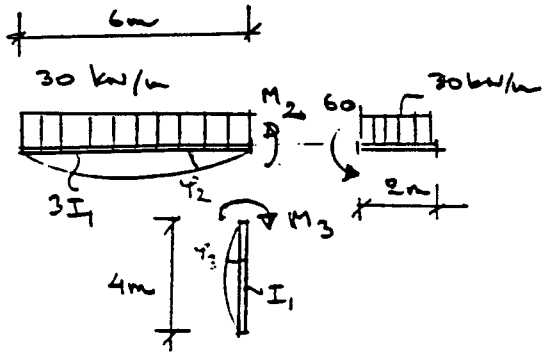
$$V_{CV} = +32,9 - 20\cdot 4\cdot\cos 20,56 = \underline{-42,0 \text{ kN}}$$

$$N_B = -32\sin 56,3 - 34,3\cdot\cos 56,3 = \underline{-45,7 \text{ kN}}$$

$$V_B = 32\cos 56,3 - 34,3\cdot\sin 56,3 = \underline{-10,8 \text{ kN}}$$

$$M^0 = \frac{20\cdot 4^2}{8} = 40 \text{ kNm} \qquad q_n = 20\cdot\cos^2 20,56 = 17,53 \text{ kN/m}$$

$$M_{AC}^{max} = \frac{32,9^2}{2\cdot 17,53} = \underline{30,9 \text{ kNm}}$$



Ex 2.19

$$\frac{6}{\cancel{3I_1 E\, 6}}\left(\frac{30 \cdot 6^2}{4} + 2M_2\right) + \frac{4}{\cancel{4EI_1}} \cdot 4M_3 = 0$$

$$270 + 2M_2 + 4M_3 = 0 \qquad \boxed{M_2 = -2M_3 - 135}$$

$$\curvearrowright \quad M_3 - M_2 - 60 = 0$$

$$M_3 + 2M_3 + 135 - 60 = 0$$

$$3M_3 = -75$$

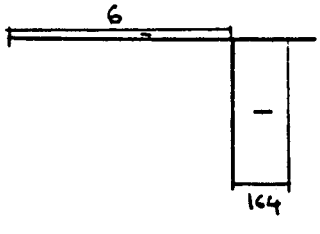
$$\underline{M_3 = -25 \text{ kNm}} \qquad M_2 = -25 - 60 = \underline{-85 \text{ kNm}}$$

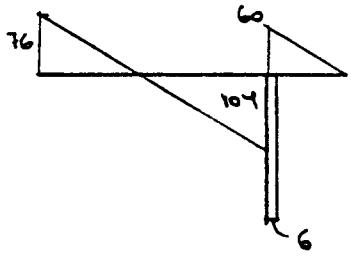
$$R_1 = \frac{30 \cdot 6}{2} - \frac{85}{6} = 90 - 14.2 = \underline{75{,}8 \text{ kN}}$$

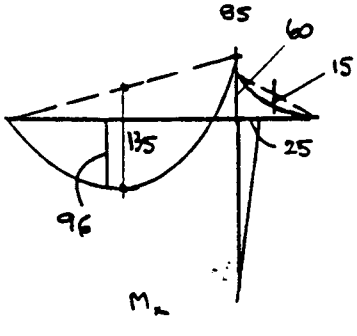
$$R_2 = 90 + 14{,}2 = \underline{104{,}2 \text{ kN}} \qquad M_0 = \frac{30 \cdot 36}{8} = \underline{135 \text{ kNm}}$$

$$R_0 = \underline{60 \text{ kN}} \qquad M_m = \frac{30 \cdot 2^2}{3} = 15$$

$$H_3 = \frac{25}{4} = 6{,}3 \text{ kN}$$

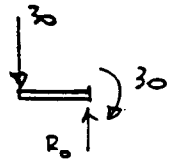


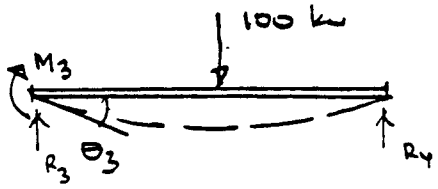


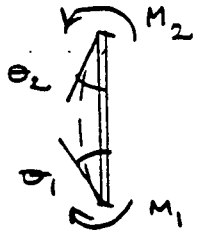


$$M_{12}^{max} = \frac{75{,}8^2}{60} = 96$$

Ex 220







$$M_3 + 30 - M_2 = 0$$

$$\boxed{M_3 = M_2 - 30}$$

$$\begin{cases} \theta_2 = \dfrac{\cancel{3}}{\cancel{6EI}}\left(-2M_2 - M_1\right) = \dfrac{\cancel{6}^{\,2}}{\cancel{6EI}}\left(2M_3 + \dfrac{3\cdot 100\cdot 6}{8}\right) \\ \theta_1 = \dfrac{3}{6EI}\left(-2M_1 - M_2\right) = 0 \end{cases}$$

$$\boxed{M_1 = -0{,}5\cdot M_2}$$

$$-2M_2 - M_1 = 4M_3 + 450$$

$$-2M_2 + 0{,}5M_2 = 4M_2 - 120 + 450$$

$$5{,}5M_2 = -330$$

$M_2 = -60$ kNm

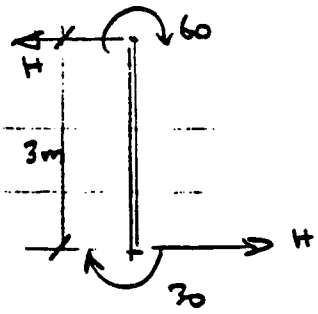
$M_1 = +30$ kNm

$M_3 = -90$ kNm

$R_0 = 30$ kN

$R_3 = 50 + \dfrac{90}{6} = 65$ kN

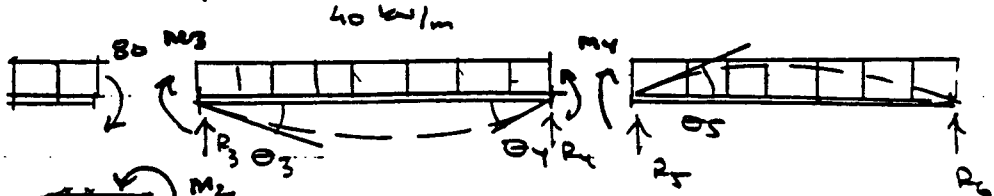
$R_4 = 50 - 15 = 35$ kN

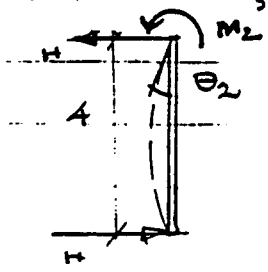


$H\cdot 3 = 90$

$H = 30$ kN

Ex 9.21





$$\circlearrowleft \quad M_2 - 80 - M_3 = 0$$

$$\boxed{M_2 = 80 + M_3}$$

$$\theta_2 = \frac{\cancel{4}}{\cancel{6EI_0}}\left(-2M_2\right) = \frac{\cancel{8}}{\cancel{2.6EI_0}}\left(2M_3 + M_4 + \frac{40 \cdot 8^2}{4}\right) = \theta_3$$

$$\theta_4 = \frac{\cancel{8}}{\cancel{2.6EI_0}}\left(2M_4 + M_3 + \frac{40 \cdot 8^2}{4}\right) = \frac{\cancel{8}}{\cancel{2.6EI_0}}\left(-2M_4 - \frac{40 \cdot 8^2}{4}\right)$$

$$\begin{cases} -2M_2 = 2M_3 + M_4 + 640 \\ 2M_4 + M_3 + 640 = -2M_4 - 640 \end{cases}$$

$$\begin{cases} -160 - 2M_3 = 2M_3 + M_4 + 640 \\ 4M_4 + M_3 + 1280 = 0 \end{cases} \qquad M_3 = -1280 - 4M_4$$

$$4M_3 + M_4 + 800 = 0$$

$$-5120 - 16M_4 + M_4 + 800 = 0 \qquad M_4 = \underline{-288 \text{ kNm}}$$

$$M_3 = \underline{-128 \text{ kNm}}$$

$$M_2 = 80 - 128 = \underline{-48 \text{ kNm}}$$

$$H \cdot 4 - 48 = 0$$

$$\underline{H = 12 \text{ kN}}$$

$$R_6 = 80 \text{ kN}$$

$$R_3 = 160 + \frac{128 - 288}{8} = 140 \text{ kN}$$

$$R_4 = 160 - 20 = 180 \text{ kN}$$

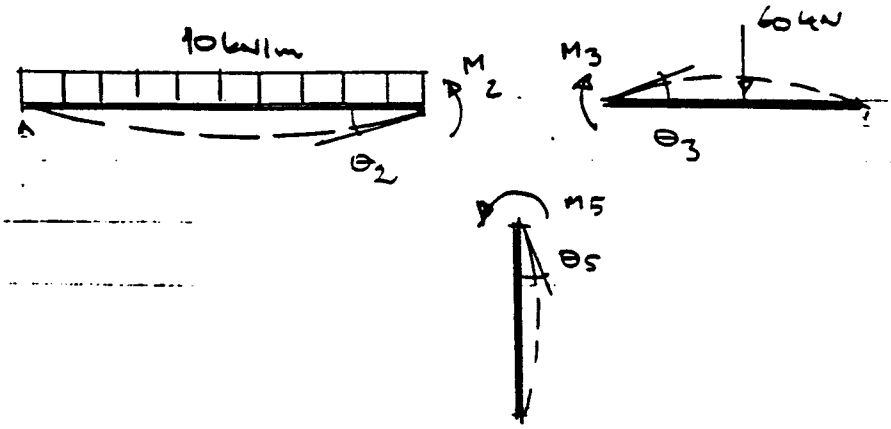
$$M_{34}^{max} = \frac{140^2}{80} - 128 = \underline{117 \text{ kNm}}$$

$$M_{56}^{max} = \frac{124^2}{80} = \underline{192 \text{ kNm}}$$

$$R_5 = 160 + \frac{288}{8} = 196 \text{ kN}$$

$$R_6 = 160 - 36 = 124 \text{ kN}$$

$$M_{34}^{o} = M_{56}^{o} = \underline{320 \text{ kNm}}$$



$$\theta_2 = \frac{12}{6EI}\left(2M_2 + \frac{10\cdot 12^2}{4}\right) \qquad \underline{M_2 + M_5 = M_3}$$

$$\theta_3 = \frac{8}{6EI}\left(-2M_3 - \frac{3\cdot 60\cdot 8}{8}\right)$$

$$\theta_5 = \frac{6}{6EI}\cdot 2M_5$$

$$\begin{cases} \cancel{12}\,(2M_2 + 360) = \cancel{12}\cdot M_5 \\ \cancel{8}\,(-2M_3 - 180) = \overset{1,5}{\cancel{12}}\, M_5 \\ M_3 = M_2 + M_5 \end{cases}$$

$$\begin{cases} 2M_2 + 360 = M_5 \\ -2M_2 - 2M_5 - 180 = 1,5M_5 \end{cases}$$

$$-2M_2 - 180 = 3,5\,(2M_2 + 360)$$

$$-1440 = 9M_2$$

$$\underline{M_2 = -160 \text{ kNm}}$$

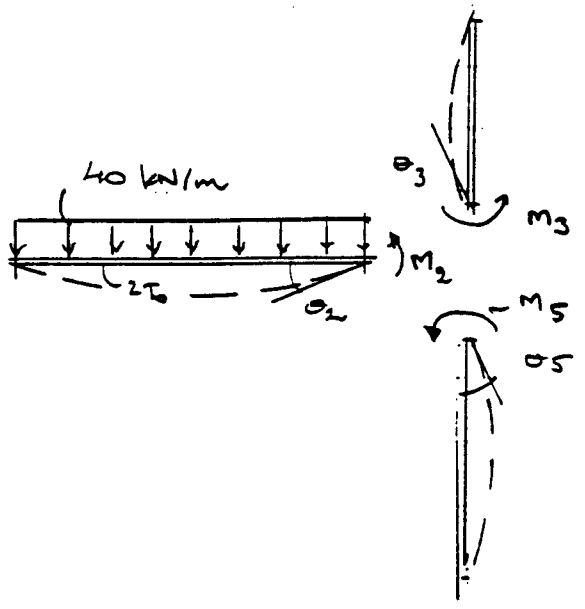
$$\underline{M_5 = -320 + 360 = 40}$$

$$\underline{M_3 = -160 + 40 = -120 \text{ kNm}}$$

$$\underline{M_{12}^{-} = \frac{10\cdot 12^2}{8} = 180}$$

$$R_1 = \frac{10\cdot 12}{2} - \frac{160}{12} = 46,7 \text{ kN} \qquad \underline{M_{12}^{max} = 109 \text{ kNm}}$$

$$R_4 = 30 - \frac{120}{8} = 15 \text{ kN} \qquad \underline{M_{34}^{max} = 15\cdot 4 = 60 \text{ kNm}}$$



$$M_2 + M_3 + M_5 = 0$$

$$\theta_2 = \frac{8}{6 \cdot 2EI_0}\left(\frac{40 \cdot 8^2}{4} + 2M_2\right)$$

$$\theta_3 = \frac{4}{6EI_0} \cdot 2M_3 \qquad \theta_5 = \frac{6}{6EI_0} \cdot 2M_5$$

$$\begin{cases} \cancel{4}(640 + 2M_2) = \cancel{4} \cdot 2M_3 \\ 8M_3 = 12M_5 \\ M_2 + M_3 + M_5 = 0 \end{cases}$$

$$\underline{M_2 = M_3 - 320}$$

$$\underline{M_5 = 0{,}667 M_3}$$

$$M_3 - 320 + M_3 + 0{,}667 M_3 = 0$$

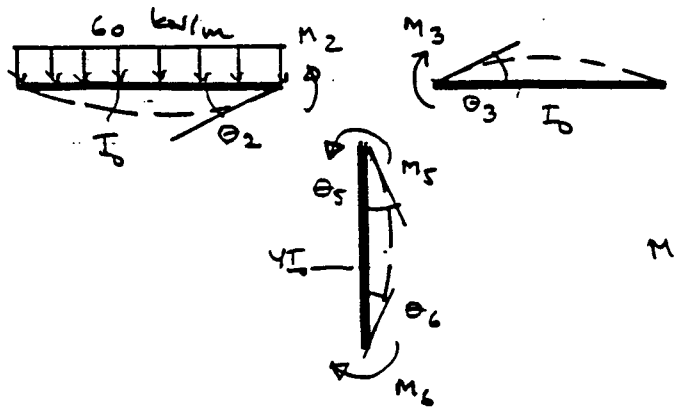
$$\underline{M_3 = 120 \text{ kNm}}$$

$$M_2 = 120 - 320 = \underline{-200 \text{ kNm}}$$

$$\underline{M_5 = 80 \text{ kNm}}$$

$$M_{12}^{0} = \frac{40 \cdot 8^2}{8} = \underline{320 \text{ kNm}}$$

$$R_1 = 40 \cdot 4 - \frac{200}{8} = 135 \qquad M_{12}^{max} = \underline{229 \text{ kNm}}$$



$$M_2 + M_5 - M_3 = 0$$

$$\theta_2 = \frac{4}{6EI_0}\cdot\left(\frac{60\cdot 4^2}{4} + 2M_2\right) \qquad \theta_3 = \frac{4}{6EI_0}\cdot(-2M_3)$$

$$\theta_5 = \frac{\cancel{4}}{\cancel{4}\cdot 6EI}\left(2M_5 + M_6\right) \qquad \theta_6 = \frac{4}{4\cdot 6EI}\left(2M_6 + M_5\right) = 0$$

$$240 + 2M_2 = -2M_3$$

$$-8\cdot M_3 = 2M_5 + M_6$$

$$\underline{M_6 = -0{,}5\cdot M_5}$$

$$\underline{M_2 = M_3 - M_5}$$

$$\begin{cases} 240 + 2M_3 - 2M_5 = -2M_3 \\ -8M_3 = 2M_5 - 0{,}5M_5 \end{cases}$$

$$\begin{cases} 240 + 4M_3 = 2M_5 \\ -8M_3 = 1{,}5M_5 \end{cases} \qquad M_5 = 2M_3 + 120$$

$$-8M_3 = 3M_3 + 180$$

$$-11M_3 = 180$$

$$M_3 = \underline{-16{,}364}$$

$$M_5 = -2\cdot 16{,}364 + 120 = \underline{+87{,}273}$$

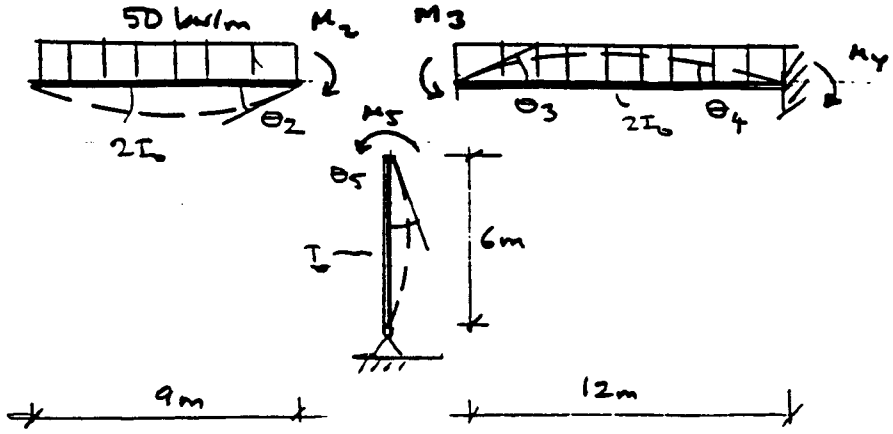
$$M_2 = -16{,}364 - 87{,}273 = \underline{-103{,}6}$$

$$\underline{M_6 = -43{,}6}$$

$$R_1 = 60\cdot 2 - \frac{103{,}6}{4} = 94{,}1 \qquad M_{12}^{max} = \underline{73{,}8 \text{ kNm}}$$

$$M_{12}^{0} = \frac{60\cdot 4^2}{8} = \underline{120 \text{ kNm}}$$

Ex 225



$$\theta_2 = \frac{9}{2.6EI_0}\left(-2M_2 + \frac{50.9^2}{4}\right)$$

$$\underline{M_2 = M_3 + M_5}$$

$$\theta_3 = \frac{12}{2.6EI_0}\left(+2M_3 - \frac{50.12^2}{4} + M_4\right)$$

$$\theta_4 = \frac{12}{2.6EI_0}\left(2M_4 + M_3 - \frac{50.12^2}{4}\right) = 0$$

$$M^0_{12} = \frac{50.9^2}{8} = 506$$

$$M^0_{34} = \frac{50.12^2}{8} = 900$$

$$\theta_5 = \frac{6}{6EI_0}\left(2M_5\right)$$

$$4.5\left(-2M_2 + 1012,5\right) = 12M_5$$

$$\cancel{6}\left(2M_3 + M_4 - 1800\right) = \cancel{12}\,2M_5$$

$$2M_4 + M_3 = 1800$$

$$\begin{cases} 4.5\left(-2M_3 - 2M_5 + 1012,5\right) = 12M_5 \\ 2M_3 + M_4 = 1800 + 2M_5 \\ 2M_4 + M_3 = 1800 \end{cases}$$

$$-9M_3 + 4556 = 21M_5 \qquad \underline{M_5 = 216,96 - 0,4286\,M_3}$$

$$2M_3 + M_4 - 2M_5 = 1800$$

$$\underline{M_4 = 900 - 0,5M_3}$$

$$2M_3 + 900 - 0,5M_3 - 433,9 + 0,857M_3 = 1800$$

$$2,357\,M_3 = 1333,9$$

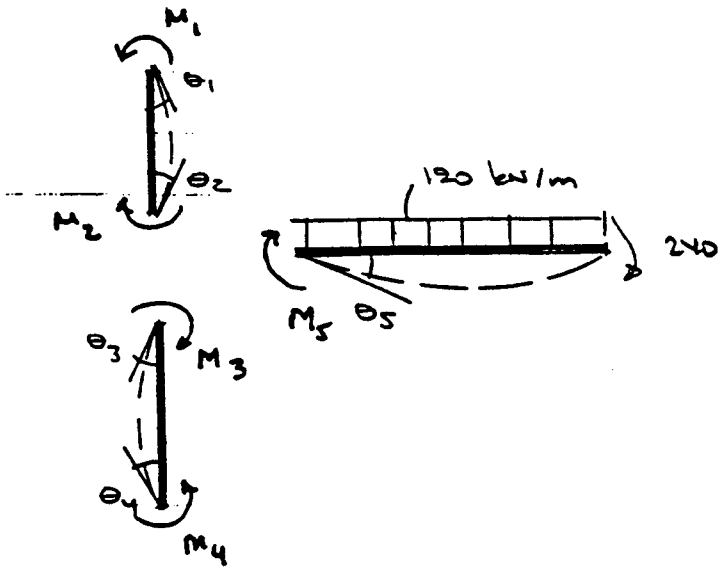
$$\underline{M_3 = 565,9 \text{ kNm}} \qquad \underline{M_4 = 617}$$

$$\underline{M_5 = -25,6 \text{ kNm}} \qquad M_2 = 565,9 - 25,6 = \underline{540 \text{ kNm}}$$

$$R_1 = 4,5 \cdot 50 - \frac{540}{9} = 165 \qquad R_4 = 6 \cdot 50 + \frac{617 - 526}{12} = 304,25$$

$$\underline{M_{12}^{max} = 272 \text{ kNm}} \qquad \underline{M_{34} = 309 \text{ kNm}}$$

8.



$M_1 = -0{,}5\,M_2 \qquad M_4 = -0{,}5\,M_3 \qquad M_2 + M_3 + M_5 = 0$

$$\theta_2 = \frac{4}{6EI}\left(2M_2 + M_1\right) = \frac{4}{6EI}\cdot 1{,}5\,M_2 = \frac{6M_2}{6EI}$$

$$\theta_3 = \frac{6}{6EI}\left(2M_3 + M_4\right) = \frac{6}{6EI}\cdot 1{,}5\,M_3 = \frac{9M_3}{6EI}$$

$$\theta_5 = \frac{6}{6EI}\left(2M_5 - 240 + \frac{120\cdot 6^2}{4}\right) = \frac{6}{6EI}\left(2M_5 + 840\right)$$

$$\begin{cases} 6M_2 = 9M_3 & M_3 = M_2\cdot 0{,}667 \\ \cancel{6}M_2 = \cancel{6}\left(2M_5 + 840\right) & M_5 = -420 + 0{,}5M_2 \end{cases}$$

$M_2 + 0{,}667\,M_2 - 420 + 0{,}5\,M_2 = 0$

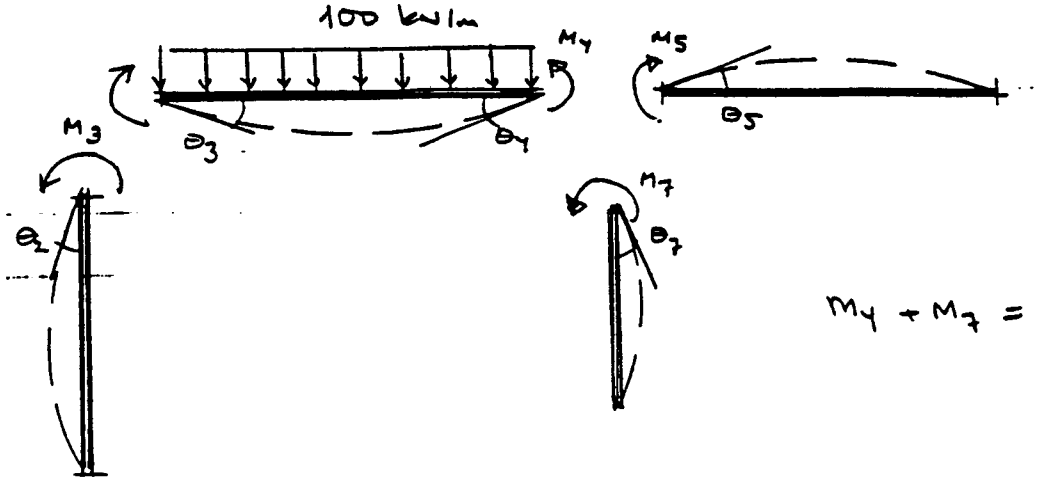
$2{,}167\,M_2 = 420$

$M_2 = \underline{193{,}8 \text{ kNm}} \qquad M_3 = \underline{129{,}2} \qquad M_5 = \underline{-323{,}1 \text{ kNm}}$

$M_1 = \underline{-96{,}9 \text{ kNm}} \qquad M_4 = \underline{-64{,}6 \text{ kNm}}$

$R_6 = 120\cdot 3 + \dfrac{240 - 323{,}1}{6} = 346 \qquad M_{56} = \underline{259 \text{ kNm}}$

$M^0_{56} = \dfrac{120\cdot 36}{8} = \underline{540 \text{ kNm}}$



$$M_4 + M_7 = M_5$$

$$\begin{cases} \theta_2 = \dfrac{7}{\cancel{6EI_0}} \cdot (-2M_3) = \dfrac{9}{2 \cdot \cancel{6EI_0}} \left(\dfrac{100 \cdot 9^2}{4} + 2M_3 + M_4 \right) \\ \dfrac{\cancel{9}}{\cancel{2 \cdot 6EI_0}} \left(\dfrac{100 \cdot 9^2}{4} + 2M_4 + M_3 \right) = \dfrac{\cancel{9}}{\cancel{2 \cdot 6EI_0}} (-2M_5) \\ \dfrac{9}{2 \cdot \cancel{6EI_0}} (-2M_5) = \dfrac{6}{\cancel{6EI_0}} (2M_7) \end{cases}$$

$$\begin{cases} -14M_3 = 91125 + 9 \cdot M_3 + 4{,}5 M_4 \\ 2025 + 2M_4 + M_3 = -2M_5 \end{cases}$$

$$-1{,}5 M_5 = 2M_7 \qquad \underline{M_7 = -0{,}75 M_5} \qquad M_5 = M_4 - 0{,}75 M_5$$

$$\underline{M_4 = 1{,}75 M_5}$$

$$\begin{cases} 0 = 91125 + 23 M_3 + 7{,}875 M_5 \\ 2025 + 5{,}5 M_5 + M_3 = 0 \end{cases} \qquad \underline{M_3 = -2025 - 5{,}5 M_5}$$

$$91125 - 46575 - 126{,}5 M_5 + 7{,}875 \cdot M_5 = 0$$

$$-374625 = 118{,}6 \, M_5$$

$$\underline{M_5 = -316 \text{ kNm}}$$

$$\underline{M_4 = -553 \text{ kNm}}$$

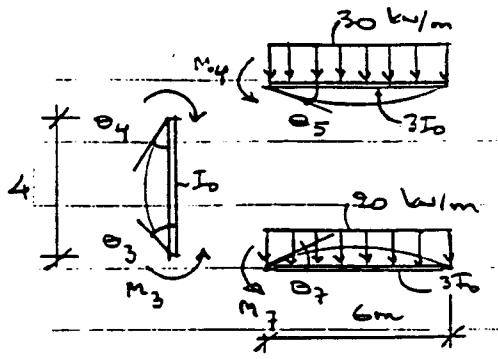
$$\underline{M_7 = +237 \text{ kNm}}$$

$$\underline{M_3 = -288 \text{ kNm}}$$

$$\underline{M^0_{34} = 1013 \text{ kNm}}$$

$$R_3 = 450 + \frac{288 - 553}{9} = 420{,}5$$

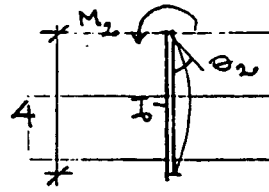
$$\underline{M^{max}_{34} = 596 \text{ kNm}}$$



$\theta_4 = \theta_5$

$\theta_2 = \theta_3 = \theta_7$ (2 eks.)

$M_2 + M_3 + M_7 = 0$



$$\theta_2 = \frac{4}{6EI_0} \cdot 2M_2$$

$$\theta_3 = \frac{4}{6EI_0}\left(2M_3 + M_4\right)$$

$$\theta_4 = \frac{4}{6EI_0}\left(2M_4 + M_3\right)$$

$$\theta_7 = \frac{6}{6 \cdot 3EI_0} = \left(-\frac{30 \cdot 6^2}{4} + 2M_7\right) \qquad \theta_5 = \frac{6}{6 \cdot 3EI_0}\left(\frac{30 \cdot 6^2}{4} - 2M_4\right)$$

$$\begin{cases} \cancel{4}\left(2M_4 + M_3\right) = \cancel{2}\left(\overset{135}{\cancel{270}} - \cancel{2}M_4\right) & (1) \\ \overset{4}{\cancel{8}}M_2 = \overset{4}{\cancel{8}}M_3 + \cancel{4}M_4 = \cancel{2}\left(-180 + 2M_7\right) & (2) \quad (3) \\ M_2 + M_3 + M_7 = 0 & (4) \end{cases}$$

(1) $\boxed{M_3 = 135 - 3M_4}$ $\qquad$ $\boxed{M_7 = 2M_2 + 90}$ (3)

$$\begin{cases} (2) \quad 4M_2 = 4\left(135 - 3M_4\right) + 2M_4 \\ (4) \quad M_2 + 135 - 3M_4 + 2M_2 + 90 = 0 \end{cases}$$

$$\begin{cases} 4M_2 = 540 - 10M_4 \\ 225 + 3M_2 - 3M_4 = 0 \end{cases} \qquad \boxed{M_4 = 75 + M_2}$$

$$4M_2 = 540 - 750 - 10M_2$$

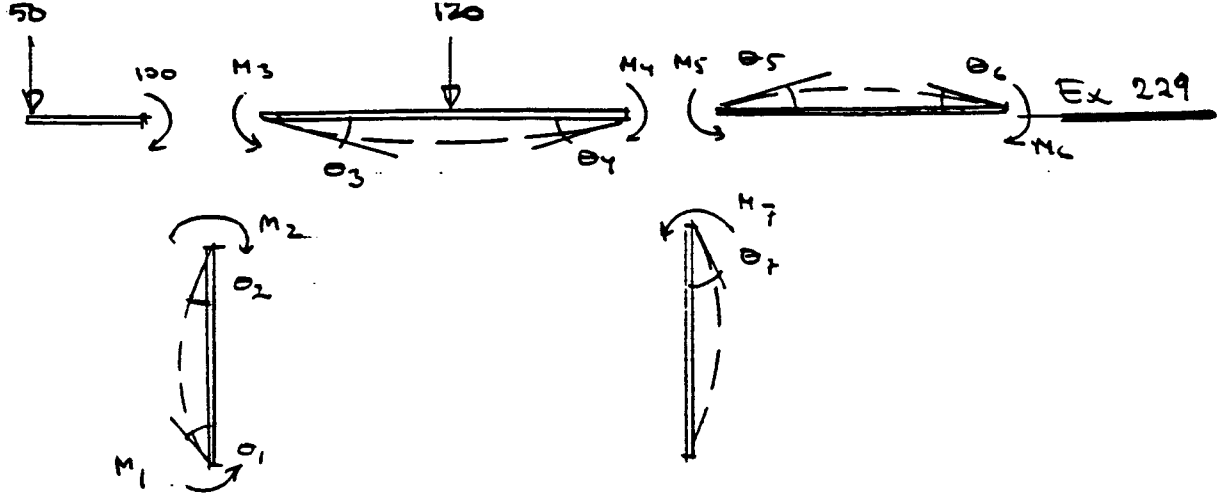
$$14M_2 = -210$$

$$M_2 = -15 \text{ kNm}$$

$$M_4 = 75 - 15 = 60 \text{ kNm}$$

$$M_3 = 135 - 3 \cdot 60 = -45 \text{ kNm}$$

$$M_7 = 2 \cdot (-15) + 90 = 60 \text{ kNm}$$



$$\theta_1 = \frac{5}{6EI}\left(2M_1 + M_2\right) = 0 \qquad \underline{M_1 = -0{,}5M_2}$$

$$\theta_2 = \frac{10}{6EI}\left(2M_6 + M_5\right) = 0 \qquad \underline{M_6 = -0{,}5M_5}$$

$$\begin{cases} 100 + M_2 = M_3 \\ M_4 = M_5 + M_7 \end{cases} \qquad \underline{M_2 = M_3 - 100}$$

$$\begin{cases} \dfrac{5}{\cancel{6EI}}\left(2M_2 - 0{,}5M_2\right) = \dfrac{8}{\cancel{6EI}}\left(\dfrac{120\cdot\cancel{8}\cdot 3}{\cancel{8}} - 2M_3 - M_4\right) \\ \dfrac{8}{\cancel{6EI}}\left(\dfrac{3\cdot 120\cdot\cancel{8}}{\cancel{8}} - 2M_4 - M_3\right) = \dfrac{5}{\cancel{6EI}}\cdot 2M_7 \\ \dfrac{5}{\cancel{6EI}}\cdot 2M_7 = \dfrac{10}{\cancel{6EI}}\left(2M_5 - 0{,}5\cdot M_5\right) \end{cases}$$

$$7{,}5\,M_2 = 2880 - 16M_3 - 8M_4$$

$$2880 - 16M_4 - 8M_3 = 10M_7$$

$$10M_7 = 15M_5 \qquad \underline{M_7 = 1{,}5\cdot M_5}$$

$$\underline{M_4 = 3{,}5M_5}$$

$$\begin{cases} 7{,}5M_3 - 750 = 2880 - 16M_3 - 20M_5 \\ 2880 - 40M_5 - 8M_3 = 15M_5 \end{cases}$$

$$\begin{cases} 23{,}5M_3 + 20M_5 = 3630 \\ 8M_3 + 55M_5 = 2880 \end{cases} \qquad \underline{M_3 = 360 - 6{,}875M_5}$$

$$8460 - 161{,}56\,M_5 + 20M_5 = 3630$$

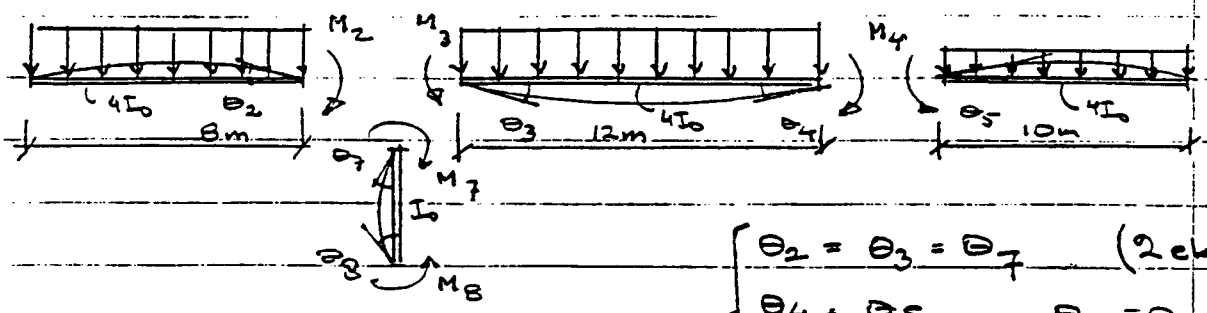
$$4830 = 141{,}56\,M_5$$

$\underline{M_5 = 34{,}1 \text{ kNm}}$ $\quad$ $\underline{M_3 = 125{,}4 \text{ kNm}}$ $\quad$ $\underline{M_4 = 85{,}3 \text{ kNm}}$

$\underline{M_7 = 51{,}2 \text{ kNm}}$ $\quad$ $\underline{M_2 = 25{,}4 \text{ kNm}}$ $\quad$ $\underline{M_1 = -12{,}7 \text{ kNm}}$

$\underline{M_6 = -17{,}1 \text{ kNm}}$ $\quad$ $R_4 = 60 + \dfrac{85{,}3 - 125{,}4}{8} = 55{,}0$

$$M_{34} = \underline{135 \text{ kNm}}$$



$$\begin{cases} \theta_2 = \theta_3 = \theta_7 \quad (2\text{ eks.}) \\ \theta_4 = \theta_5 \qquad \theta_8 = 0 \\ M_2 - M_3 + M_7 = 0 \end{cases}$$

$$\begin{cases} \dfrac{\cancel{8}^{\,4}}{\cancel{6\cdot 4EI_0}}\left(2M_2 - \dfrac{30\cdot 8^2}{4}\right) = \dfrac{\cancel{12}^{\,6}}{\cancel{6\cdot 4EI_0}}\left(-2M_3 - M_4 + \dfrac{30\cdot 12^2}{4}\right) = \dfrac{\cancel{4}^{\,2}}{\cancel{6EI_0}}\left(2M_7 + M_8\right) \\ \left(-2M_4 - M_3 + \dfrac{30\cdot 12^2}{4}\right)\dfrac{12}{6\cdot 4EI_0} = \left(2M_4 - \dfrac{20\cdot 10^2}{4}\right)\cdot\dfrac{10}{6\cdot 4EI_0} \\ M_2 - M_3 + M_7 = 0 \qquad 2M_8 + M_7 = 0 \qquad \boxed{M_8 = -0{,}5M_7} \end{cases}$$

$$2M_2 - 480 = 1{,}5\left(-2M_3 - M_4 + 1080\right) = 3\cdot M_7 \qquad \boxed{M_7 = \frac{2M_2}{3} - 160}$$

$$\left(-2M_4 - M_3 + 1080\right)\cdot 1{,}2 = 2M_4 - 500$$

$$M_2 - M_3 + M_7 = 0 \qquad \boxed{M_3 = M_2 + \frac{2M_2}{3} - 160 = \frac{5M_2}{3} - 160}$$

$$\begin{cases} 2M_2 - 480 = -3M_3 - 1{,}5M_4 + 1620 \\ -2{,}4M_4 - 1{,}2M_3 + 1296 = 2M_4 - 500 \end{cases}$$

$$\begin{cases} 2M_2 + 3\left(\dfrac{5M_2}{3} - 160\right) + 1{,}5M_4 = 2100 \\ -4{,}4M_4 - 1{,}2\left(\dfrac{5M_2}{3} - 160\right) + 1796 = 0 \end{cases}$$

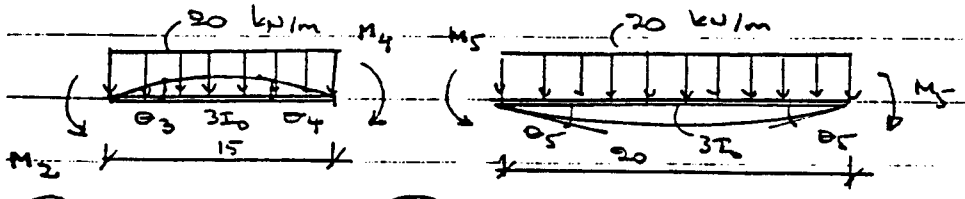
$$\begin{cases} 7M_2 + 1{,}5M_4 = 2580 \\ -4{,}4M_4 - 2M_2 + 1908 = 0 \qquad \boxed{M_2 = 994 - 2{,}2M_4} \end{cases}$$

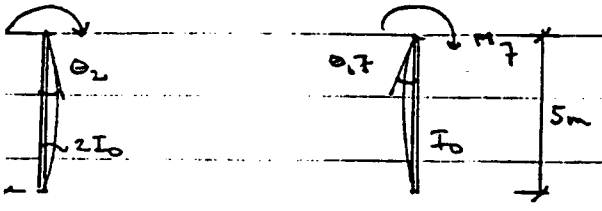
$$6958 - 15{,}4M_4 + 1{,}5M_4 = 2580 \qquad \underline{M_4 = 315 \text{ kNm}}$$

$$M_2 = 994 - 2{,}2\cdot 315 = \underline{301 \text{ kNm}}$$

$$M_3 = \frac{5\cdot 301}{3} - 160 = \underline{342 \text{ kNm}}$$

$$M_7 = \frac{2\cdot 301}{3} - 160 = \underline{41 \text{ kNm}} \qquad M_8 = -0{,}5\cdot 41 = \underline{-20 \text{ kNm}}$$





$$\begin{cases} \theta_2 = \theta_3 \\ \theta_4 = \theta_5 = \theta_7 \quad (2 \text{ eqns}) \\ M_4 - M_5 + M_7 = 0 \end{cases}$$

$$\frac{5}{\cancel{6 \cdot 2I_0}} \cdot (-2M_2) = \frac{15}{\cancel{6 \cdot 3I_0}} \left(2M_2 + M_4 - \frac{20 \cdot 15^2}{4} \right)$$

$$\frac{\cancel{15}^{3}}{\cancel{6 \cdot 3I_0}} \left(2M_4 + M_2 - \frac{20 \cdot 15^2}{4} \right) = \frac{\cancel{20}^{4}}{\cancel{6 \cdot 3I_0}} \left(-2M_5 - M_5 + \frac{20 \cdot 20^2}{4} \right) = \frac{\cancel{5}^{3}}{\cancel{6I_0 \cdot 3}} \cdot (-2M_7)$$

$$M_4 - M_5 + M_7 = 0$$

$$\begin{cases} -5M_2 = 10M_2 + 5M_4 - 5625 \quad \text{o.k.} \\ 6M_4 + 3M_2 - 3375 = -8M_5 - 4M_5 + 8000 = 6M_7 \\ M_4 - M_5 + M_7 = 0 \end{cases}$$

$$\boxed{M_7 = -2M_5 + 1333} \quad \text{o.k.}$$

$$M_4 - M_5 - 2M_5 + 1333 = 0$$

$$\boxed{M_5 = \frac{M_4}{3} + 444{,}4} \quad \text{ok}$$

$$\begin{cases} -15M_2 = 5M_4 - 5625 \quad \text{ok} \\ 6M_4 + 3M_2 - 3375 = -12\left(\frac{M_4}{3} + 444{,}4 \right) + 8000 \end{cases}$$

$$\boxed{M_4 = -3M_2 + 1125} \quad \text{o.k.}$$

$$= -4M_4 - 5333 + 8000$$

$$10M_4 + 3M_2 = 6042$$

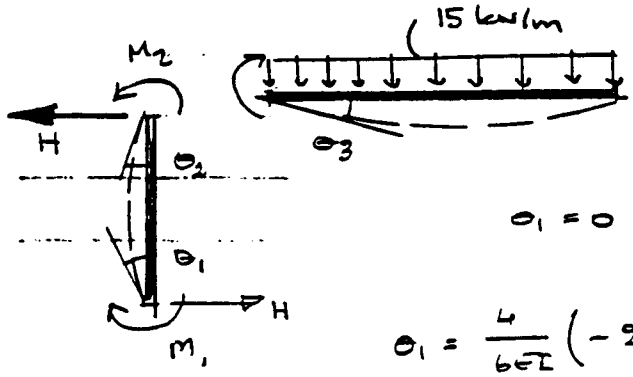
$$-30M_2 + 11250 + 3M_2 = 6042$$

$$27M_2 = 5208 \qquad \underline{M_2 = 193 \text{ kNm}}$$

$$M_4 = -3 \cdot 193 + 1125 = \underline{546 \text{ kNm}}$$

$$M_5 = 546/3 + 444{,}4 = \underline{626 \text{ kNm}}$$

$$M_7 = -2 \cdot 626 + 1333 = \underline{80 \text{ kNm}}$$



$\theta_1 = 0 \qquad \theta_2 = \theta_3$

$$\theta_1 = \frac{4}{6EI}\left(-2M_1 - M_2\right) = 0 \qquad \underline{M_1 = -0{,}5\,M_2}$$

$$\frac{4}{\cancel{6EI}}\left(-2M_2 - M_1\right) = \frac{10}{\cancel{6EI}}\left(2M_2 + \frac{15\cdot 10^2}{4}\right)$$

$$4\left(-1{,}5\,M_2\right) = 10\left(2M_2 + 375\right)$$

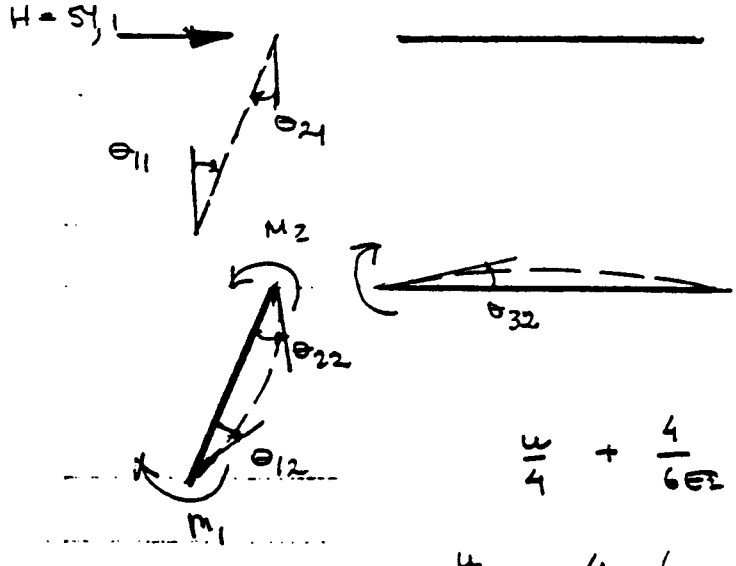
$$-0{,}6\,M_2 = 2M_2 + 375$$

$$\underline{M_2 = -144{,}2 \text{ kN}} \qquad \underline{M_1 = +72{,}1 \text{ kN}}$$

FRIRAM

$$\circlearrowleft \; H\cdot 4 - 144{,}2 - 72{,}1 = 0$$

$$\underline{H = 54{,}1 \text{ kN}}$$



$$\theta_{11} + \theta_{12} = 0$$

$$-\theta_{21} + \theta_{22} = \theta_{32}$$

$$\frac{u}{4} + \frac{4}{6EI}\left(2M_1 + M_2\right) = 0$$

$$-\frac{u}{4} + \frac{4}{6EI}\left(2M_2 + M_1\right) = -\frac{10}{6EI}\cdot 2M_2$$

$$\frac{4}{\cancel{6EI}}\left(2M_1 + M_2\right) + \frac{4}{\cancel{6EI}}\left(2M_2 + M_1\right) = -\frac{20M_2}{\cancel{6EI}}$$

$$12M_1 + 12M_2 = -20M_2 \qquad \underline{M_2} = -\frac{12M_1}{32} = -0{,}375\,M_1$$

$$\circlearrowright \; 54{,}1\cdot 4 + M_1 - M_2 = 0 \qquad \underline{M_2 = +59{,}0 \text{ kNm}}$$

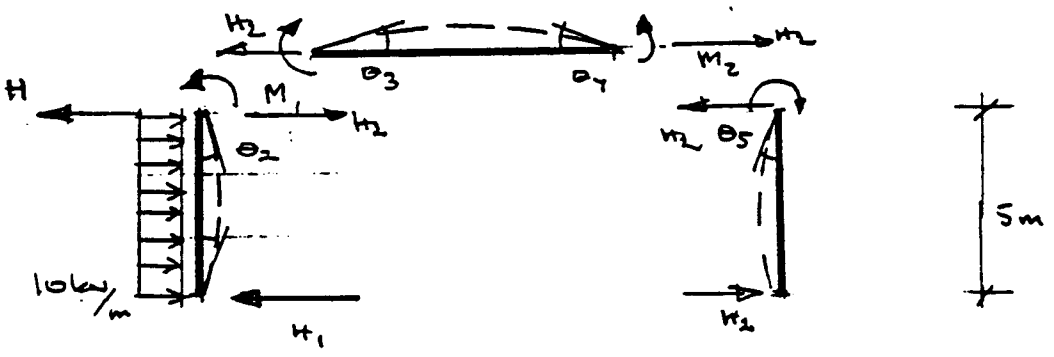
$$216{,}3 + M_1 + 0{,}375\,M_1 = 0$$

$$\underline{M_1 = -157{,}3 \text{ kNm}}$$

SAMMANSTÄLLN

$$M_1^{tot} = +72{,}1 - 157{,}3 = \underline{-85{,}2}$$

$$M_2^{tot} = -144{,}2 + 59{,}0 = \underline{-85{,}2}$$



FASTHÅLLEN

$$\begin{cases} \dfrac{5}{6EI}\left(\dfrac{10\cdot 5^2}{4} + 2M_1\right) = \dfrac{12}{6EI}\left(-2M_1 - M_2\right) \\ \dfrac{12}{6EI}\left(-2M_2 - M_1\right) = \dfrac{5}{6EI}\left(2M_2\right) \end{cases}$$

$$\begin{cases} 62{,}5 + 2M_1 = -4{,}8M_1 - 2{,}4M_2 \\ -4{,}8M_2 - 2{,}4M_1 = 2M_2 \end{cases} \qquad M_1 = -\frac{6{,}8}{2{,}4}M_2 = -2{,}833\,M_2$$

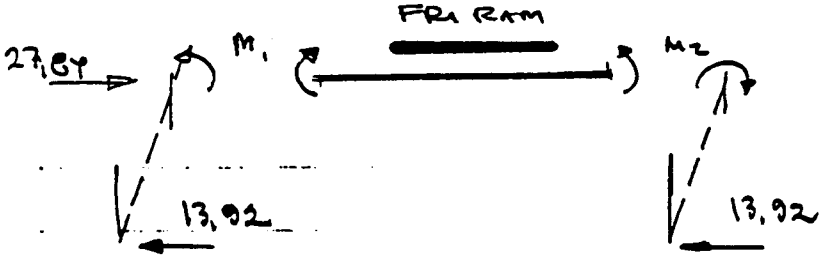
$$62{,}5 - 19{,}267\,M_2 + 2{,}4M_2 = 0 \qquad M_2 = +3{,}7055$$

$$M_1 = -10{,}5 \text{ kNm}$$

$$H_2 = \frac{3{,}7}{5} = 0{,}74 \text{ kN}$$

$$\circlearrowleft \; 10{,}5 + 0{,}74\cdot 5 - H\cdot 5 + 10\cdot 5\cdot 2{,}5 = 0 \qquad H = 27{,}84 \text{ kN}$$

FRI RAM



$$M_1 = 13{,}92\cdot 5 = +69{,}6 \text{ kNm}$$

$$M_2 = -69{,}6 \text{ kNm}$$

SAMMANSTÄLLNING

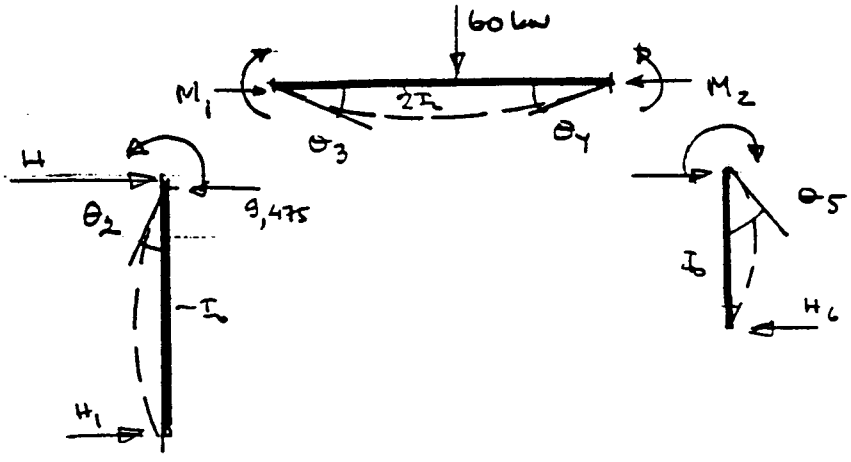
$$M_1^{tot} = -10{,}5 + 69{,}6 = +59{,}1 \text{ kNm} \qquad M_0 = \frac{10\cdot 5^2}{8} = 31$$

$$M_2^{tot} = +3{,}7 - 69{,}6 = -65{,}9 \text{ kNm}$$

12,68 $\quad H_1 = \dfrac{10\cdot 5}{2} + \dfrac{59{,}1}{5} = 36{,}82 \qquad M_{max} = 67{,}8$ kNm

Ex 234

FASTHÅLLEN



$$H_6 = \frac{37,9}{4} = 9,475$$

$$\frac{6}{\cancel{6EI_0}}\left(-2M_1\right) = \frac{\overset{4}{\cancel{8}}}{\cancel{2\cdot 6EI_0}}\left(2M_1 + M_2 + \frac{3\cdot 60\cdot \cancel{8}}{\cancel{8}}\right)$$

$$\frac{\cancel{8}}{\cancel{2\cdot 6EI_0}}\left(2M_2 + M_1 + \frac{3\cdot 60\cdot \cancel{8}}{\cancel{8}}\right) = \frac{\cancel{4}}{\cancel{6EI_0}}\cdot\left(-2M_2\right)$$

$$\begin{cases} -3M_1 = 2M_1 + M_2 + 180 \\ 2M_2 + M_1 + 180 = -2M_2 \end{cases}$$

$$\boxed{M_1 = -4M_2 - 180}$$

$$0 = -20M_2 - 900 - M_2 + 180$$

$$19M_2 = -720$$

$$\underline{M_2 = -37,9 \text{ kNm}}$$

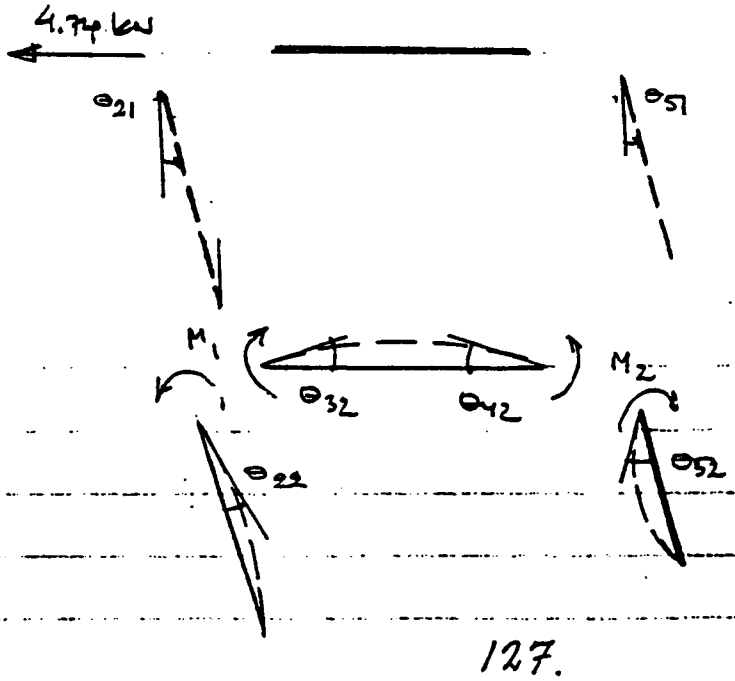
$$\underline{M_1 = -28,4 \text{ kNm}}$$

R

$$\curvearrowright \quad 28,4 + (H - 9,475)\cdot 6 = 0$$

$$\underline{H = 4,74 \text{ kN}}$$

FRI RAM



$$\theta_{32} = \theta_{22} + \theta_{21}$$

$$\theta_{42} = \theta_{52} - \theta_{51}$$

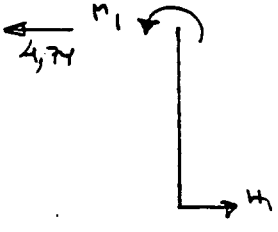
forts →

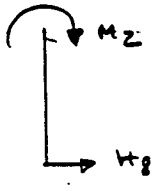
$$\begin{cases} \dfrac{u}{6} + \dfrac{6}{6EI}\left(+2M_1\right) = \dfrac{\cancel{8}4}{\cancel{2}\cdot 6EI}\left(-2M_1 - M_2\right) \\[2ex] \dfrac{\cancel{8}4}{\cancel{2}\cdot 6EI}\left(-2M_2 - M_1\right) = \dfrac{4}{6EI}\cdot 2M_2 - \dfrac{u}{4} \end{cases}$$

$$\begin{cases} u + \dfrac{72\,M_1}{6EI} = \dfrac{-48\,M_1 - 24\,M_2}{6EI} \\[2ex] \dfrac{-32M_2 - 16M_1}{6EI} = \dfrac{32M_2}{6EI} - u \end{cases}$$

$$-120\,M_1 - 24\,M_2 = 64\,M_2 + 16\,M_1$$

$$-88\,M_2 = 136\,M_1 \qquad \boxed{M_1 = -0{,}647\,M_2}$$





$$H_1 = -\frac{M_1}{6} \qquad H_8 = \frac{M_2}{4}$$

$$4{,}74 + \frac{M_1}{6} - \frac{M_2}{4} = 0$$

$$28.44 + M_1 - 1.5\,M_2 = 0$$

$$28.44 - 0{,}647\,M_2 - 1.5\,M_2 = 0$$

$$\underline{M_2 = +13{,}2 \text{ kNm}}$$

$$\underline{M_1 = -8{,}6 \text{ kNm}}$$

SAMMANSTÄLLNING

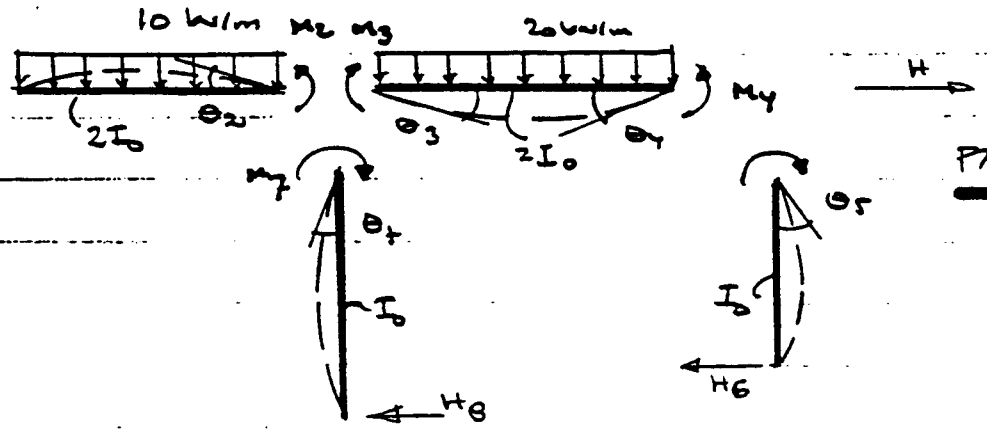
$$M_1 = -28{,}4 - 8{,}6 = \underline{-37{,}0 \text{ kNm}}$$

$$M_2 = -37{,}9 + 13{,}2 = \underline{-24{,}7 \text{ kNm}}$$

$$R_4 = 30 + \frac{24{,}7 - 37{,}0}{8} = 28{,}46 \qquad M_{fält} = 28{,}46\cdot 4 - 24{,}7 = 89$$

Ex 235

PASTHÅLLEN



$$\frac{\cancel{8}^{\,4}}{\cancel{6\,2EI_0}}\left(-2M_2 - \frac{10\cdot 8^2}{4}\right) = \frac{6}{\cancel{6EI_0}}\,2M_7$$

$$\frac{6}{\cancel{6EI_0}}\cdot 2M_7 = \frac{\cancel{8}\,4}{\cancel{6\,2EI_0}}\left(2M_3 + M_4 + \frac{20\cdot 8^2}{4}\right)$$

$$\frac{\cancel{8}}{\cancel{6\cdot 2EI_0}}\left(2M_4 + M_3 + \frac{20\cdot 8^2}{4}\right) = \frac{4}{\cancel{6EI_0}}\cdot(-2M_4)$$

2, 7 $\quad -2M_2 - 160 = 3M_7$ $\qquad \underline{M_2 = -1{,}5\,M_7 - 80}$

3, 4, 7 $\quad 3M_7 = 2M_3 + M_4 + 320$

3, 4 $\quad 2M_4 + M_3 + 320 = -2M_4$ $\qquad \underline{M_3 = -4M_4 - 320}$

2, 3, 7 $\quad M_2 = M_3 + M_7$

$$\begin{cases} 3M_7 = -8M_4 - 640 + M_4 + 320 \\ -1{,}5M_7 - 80 = -4M_4 - 320 + M_7 \end{cases}$$

$$\begin{cases} 3M_7 + 7M_4 = -320 \\ 4M_4 = 2{,}5M_7 - 240 \end{cases} \qquad \underline{M_7 = 96 + 1{,}6\,M_4}$$

$$288 + 4{,}8\,M_4 + 7M_4 = -320$$

$$11{,}8\,M_4 = -608 \qquad \underline{M_4 = -51{,}53}$$

$$\underline{M_7 = 96 - 1{,}6\cdot 51{,}53 = +13{,}56}$$

$$\underline{M_3 = -113{,}90}$$

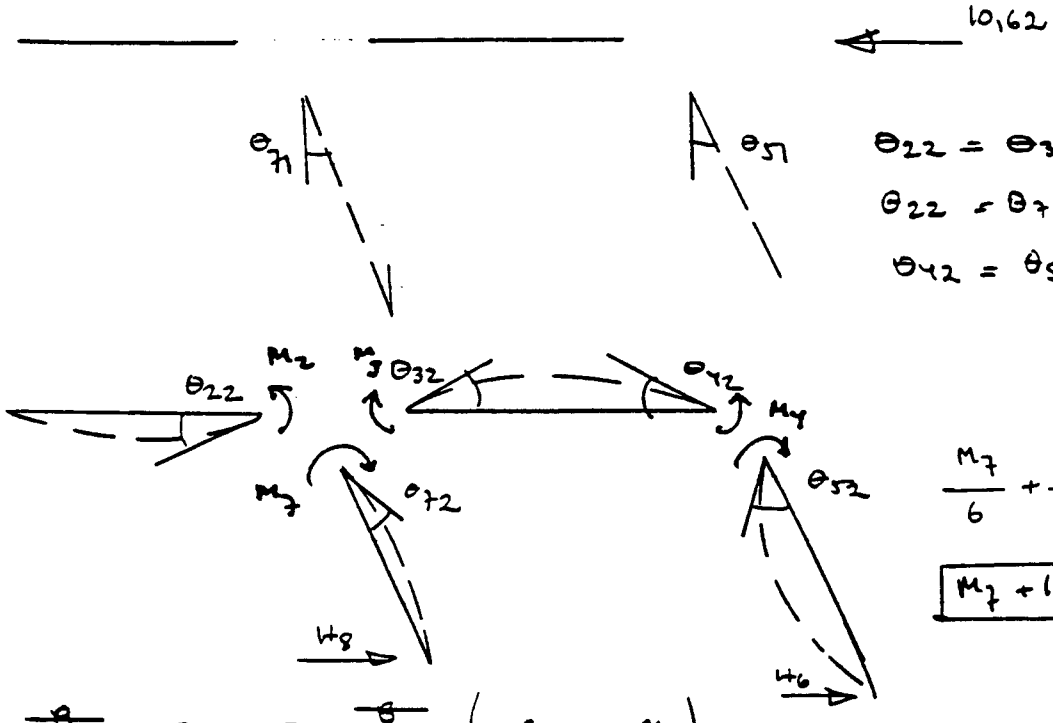
$$\underline{M_2 = -109{,}34}$$

$$H_8 = -\frac{13{,}56}{6}$$

$$H_6 = \frac{51{,}53}{4} \qquad \underline{H = 19{,}62 \text{ åt höger}}$$

forts →

Per asm



$\theta_{22} = \theta_{32}$

$\theta_{22} = \theta_{71} + \theta_{72}$

$\theta_{42} = \theta_{52} - \theta_{51}$

$$\frac{M_7}{6} + \frac{M_4}{4} = 10{,}62$$

$$\boxed{M_7 + 1{,}5 M_4 = 63{,}7}$$

$$\begin{cases} \dfrac{8}{6 \cdot 2EI_0} \cdot 2M_2 = \dfrac{8}{6 \cdot 2EI_0}\left(-2M_3 - M_4\right) \\[2ex] \dfrac{8 \cdot 6}{6 \cdot 2EI_0} \cdot 2M_2 = \dfrac{u}{6} - \dfrac{6 \cdot 6}{6EI_0} \cdot 2M_7 \qquad u = \dfrac{(8M_2 + 12M_7)6}{6 \cdot EI_0} \\[2ex] \dfrac{8 \cdot 4^2}{6 \cdot 2EI_0}\left(-2M_4 - M_3\right) = \dfrac{4 \cdot 4}{6EI_0} \cdot 2M_4 - \dfrac{u}{4} \qquad u = \dfrac{+32M_4 + 32M_4 + 16M_3}{6EI_0} \\[2ex] M_2 = M_3 + M_7 \end{cases}$$

$$\begin{cases} 48M_2 + 72M_7 = +64M_4 + 16M_3 \\ 2M_2 + 2M_3 + M_4 = 0 \\ M_2 = M_3 + M_7 \end{cases}$$

$$\begin{cases} 48 \cdot M_3 + 48M_7 + 72M_7 = +64M_4 + 16M_3 \\ 2M_3 + 2M_7 + 2M_3 + M_4 = 0 \end{cases}$$

$$\begin{cases} 32M_3 + 120M_7 - 64M_4 = 0 \\ 4M_3 + M_4 + 2M_7 = 0 \end{cases} \qquad 32M_3 = -8M_4 - 16M_7$$

$$-8M_4 - 16M_7 + 120M_7 - 64M_4 = 0$$

$$-72M_4 + 104M_7 = 0$$

$$-72M_4 + 6628 - 156M_4 = 0$$

$$6628 = 228M_4 \qquad M_4 = +29{,}07 \text{ kNm}$$

fortsæt →

$M_7 = +20{,}12$ kNm

$M_3 = -17{,}32$ kNm

$M_2 = +2{,}80$ kNm

SAMMANSTÄLLNING :

$M_2 = -100{,}34 + 2{,}80 = -97{,}5$

$M_3 = -113{,}90 - 17{,}32 = -131{,}2$ Kollat !

$M_4 = -57{,}53 + 29{,}07 = -225$

$M_7 = +13{,}56 + 20{,}12 = +33{,}7$

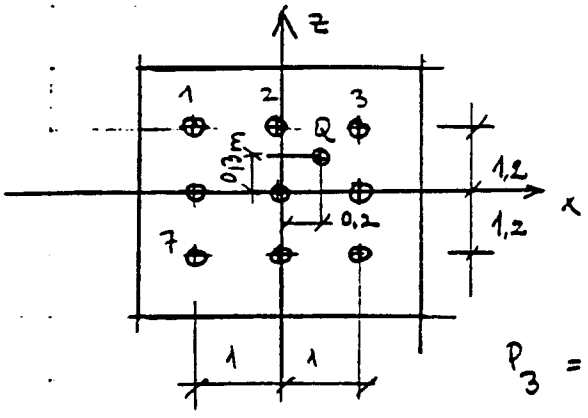
$M^0_{12} = 80$

$M^0_{34} = \frac{20 \cdot 8^2}{8} = 160$

$R_1 = 10 \cdot 4 - \frac{97{,}5}{8} = 27{,}8$ $\quad M_{12}^{max} = 38{,}7$ kNm

$R_4 = 20 \cdot 4 + \frac{225 - 131{,}2}{8} = 66{,}4$ $\quad M_{34}^{max} = 87{,}8$

Ex 236



$M_z = 1900 \cdot 0{,}2 = 380$ kNm

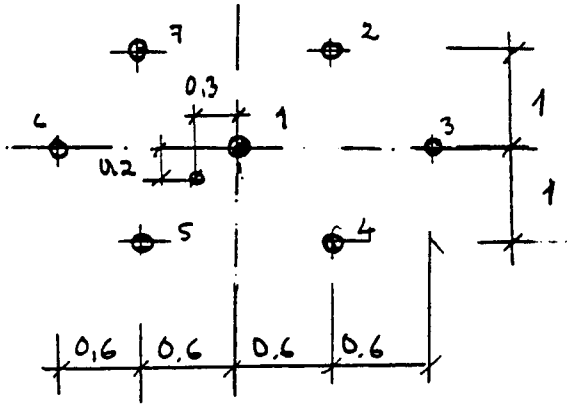
$M_x = 1900 \cdot 0{,}3 = 570$ kNm

$\Sigma r_x^2 = 6 \cdot 1^2 = \underline{6\,m^2}$ $\quad \Sigma r_z^2 = 6 \cdot 1{,}2^2 = \underline{8{,}64\,m^2}$

$$P_3 = \frac{1900}{9} + \frac{380}{6} \cdot 1 + \frac{570}{8{,}64} \cdot 1{,}2 = \underline{353\text{ kN}}$$

$$P_7 = 211 - 63 - 79 = \underline{69\text{ kN}}$$

Ex 237



$\Sigma r_x^2 = 4 \cdot 0{,}6^2 + 2 \cdot 1{,}2^2 = \underline{4{,}32\,m^2}$

$\Sigma r_z^2 = 4 \cdot 1^2 = \underline{4\,m^2}$

$M_z = 1900 \cdot 0{,}3 = 570$ kNm

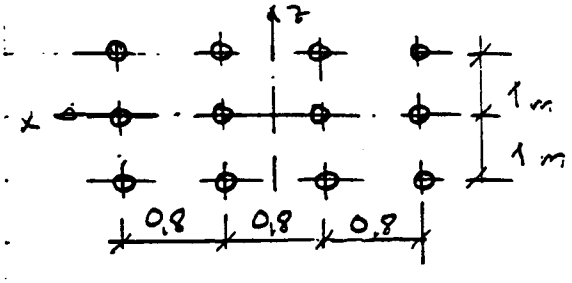
$M_x = 1900 \cdot 0{,}2 = 380$ kNm

$$P_5 = \frac{1900}{7} + \frac{570}{4{,}32} \cdot 0{,}6 + \frac{380}{4} \cdot 1 = \underline{446\text{ kN}}$$

$$\left(P_6 = \frac{1900}{7} + \frac{570}{4{,}32} \cdot 1{,}2 = 430\text{ kN}\right)$$

$$P_2 = 271{,}4 - 79{,}2 - 95 = \underline{97\text{ kN}}$$

Ex 238



$\Sigma r_x^2 = 6 \cdot 0{,}4^2 + 6 \cdot 1{,}2^2 = \underline{9{,}6\,m^2}$

$\Sigma r_z^2 = 8 \cdot 1^2 = \underline{8\,m^2}$

$M_z = 1800 \cdot e_x \quad M_x = 1800 \cdot 0{,}2 = 360$

$$P_{max} = \frac{1800}{12} + \frac{1800 \cdot e_x \cdot 1{,}2}{9{,}6} + \frac{360 \cdot 1}{8} = 330 \qquad e_x = 0{,}6\,m$$

$$P_{min} = \frac{1800}{12} - \frac{1800 \cdot e_x \cdot 1{,}2}{9{,}6} - \frac{360 \cdot 1}{8} = 0 \qquad \underline{e_x = 0{,}47\,m}$$

ÅVGÖRANDE!

$3 \cdot 0 + 1 \cdot 1 + 1 \cdot 2 = 5 \cdot e$

$e = 0.6\,m$

$\tan\alpha = 0{,}25$

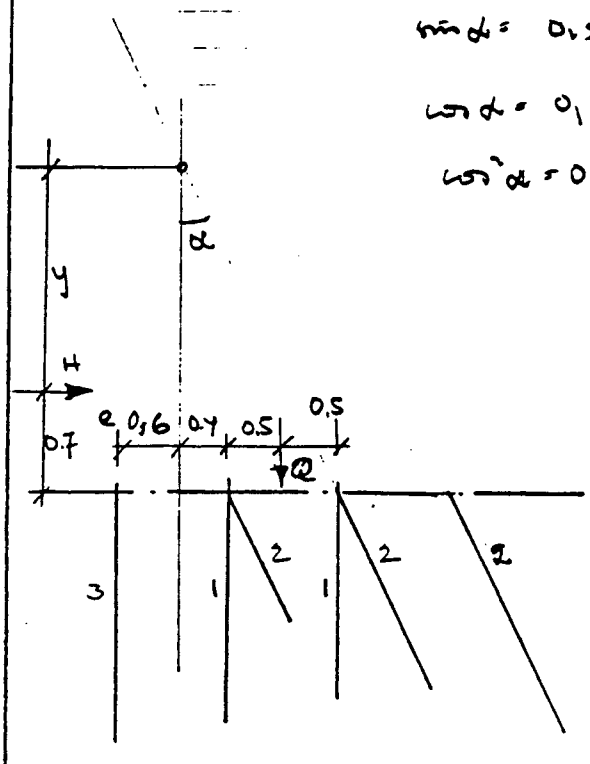
$\sin\alpha = 0{,}24$

$\cos\alpha = 0{,}97$

$\cos^2\alpha = 0{,}94$

$z_0 = 4 \cdot 1.4 = 5{,}6\,m$

$\curvearrowright \quad M = 2600 \cdot 0.9 - 400 \cdot 4.9 = 380$



$\Sigma r^2 = 3 \cdot 0.6^2 + 1 \cdot 0.4^2 + 1 \cdot 1.4^2 + 4 \cdot 1^2 \cdot 0{,}94 = 6{,}96^2$

Av $Q = 2800$ kN erhålls

$P_V = \frac{2600}{5} = 520$ kN $\quad P_S = 0$

Av $H = 400$ kN erhålls

$P_V = -\frac{400}{5 \cdot \tan\alpha} = -\frac{400}{5 \cdot 0{,}25} = -320$ kN

$P_S = +\frac{400}{6 \cdot 0.24} = 278$ kN

Av M erhålls

$P_i = \frac{380}{6{,}96} r_i = 54{,}6 \cdot r_i$

VERTIKALK

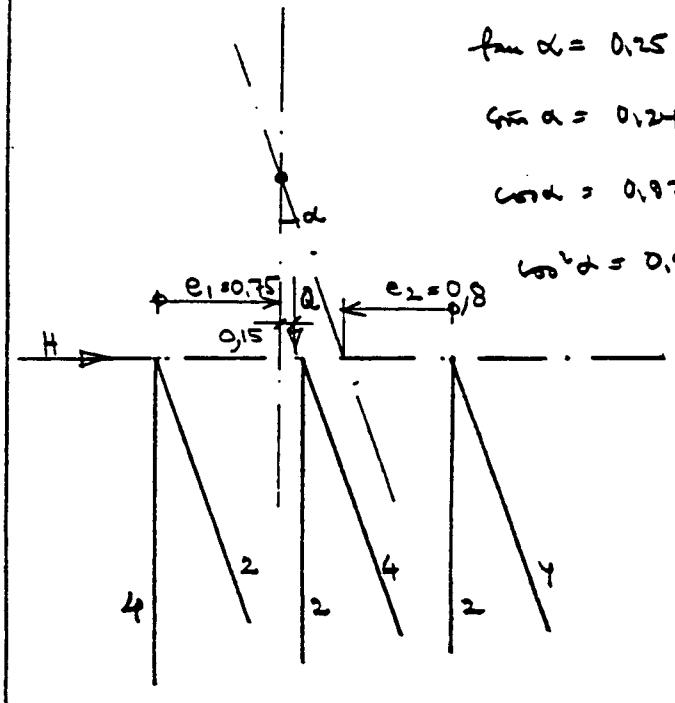
Av \ Pāle	a	b_2	c_2	b_{13}	c_{13}	d
Q	520	520	520	—	—	—
H	-320	-320	-320	+278	+278	+278
M	-33	+22	+76	-53	0	+53
Σ	+167	+222	+276	+225	+278	+331

$4\cdot 0 + 2\cdot 1 + 2\cdot 2 = 8 e_1$

$e_1 = 0{,}75 m$

$4\cdot 0 + 4\cdot 1 + 2\cdot 2 = 10 e_2$

$e_2 = 0.8 m$



$\tan\alpha = 0{,}25$

$\sin\alpha = 0{,}24$

$\cos\alpha = 0{,}97$

$\cos^2\alpha = 0{,}94$

$y = 4(0.25 + 0.20) = 1{,}8 m$

$\Sigma r_i^2 = 4\cdot 0{,}75^2 + 2\cdot 0.25^2 + 2\cdot 1.25^2 +$

$+ (2\cdot 1.2^2 + 4\cdot 0{,}2^2 + 4\cdot 0{,}8^2)0{,}94 = 10{,}76$

$M = 5600\cdot 0{,}15 - 530\cdot 1.8 = -114$

Av Q erhålls

$P_V = \frac{5600}{8} = 700 kN - P_S = 0$

Av H erhålls

$P_V = -\frac{530}{8\cdot 0{,}25} = -265 kN$

$P_S = +\frac{530}{10\cdot 0{,}24} = +221 kN$

Av $M = -420$ erhålls

$P_i = \frac{-114}{10{,}76}\cdot r_i = -10{,}6 r_i$

VERTIKALA

Av \ Pålar	a_{2345}	b_{25}	c_{25}	a_{16}	b_{1346}	c_{1346}
Q	700	700	700	—	—	—
H	-265	-265	-265	+221	+221	+221
M	+8	-3	-13	+19	+9	-8
Σ	443	432	422	233	223	213

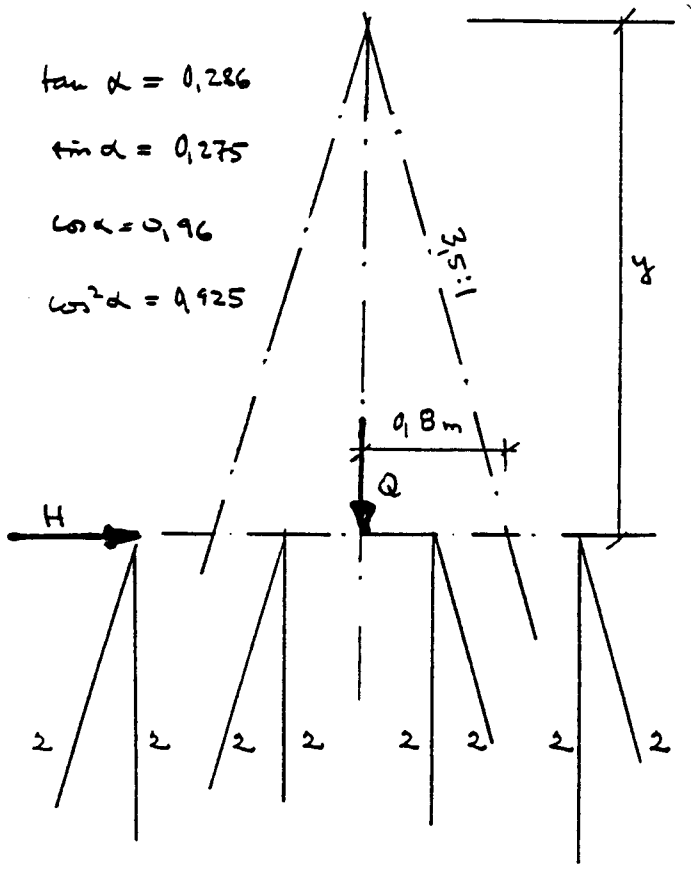
Ex 241

$\tan\alpha = 0{,}286$

$\sin\alpha = 0{,}275$

$\cos\alpha = 0{,}96$

$\cos^2\alpha = 0{,}925$



$y = 3{,}5 \cdot 0{,}8 = 2{,}8\ m$

$M = -50 \cdot 2{,}8 = -140\ kNm$

$\Sigma r^2 = 8 \cdot 0{,}4^2 \cdot 0{,}925 + 4 \cdot 0{,}4^2 + 4 \cdot 1{,}2^2 = 7{,}58\ m^2$

Q ger

$$P_V = \frac{3200}{8 + 8 \cdot 0{,}925} = 208\ kN$$

$$P_S = 208 \cdot 0{,}96 = 200\ kN$$

H ger

$P_V = 0$

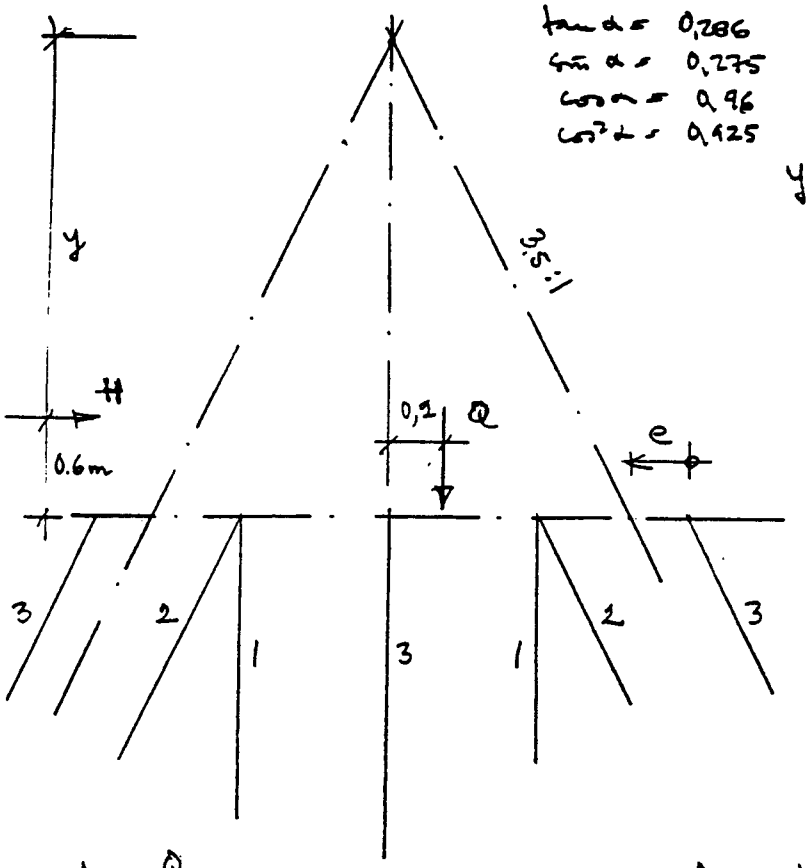
$$P_S = \pm \frac{50}{8 \cdot 0{,}275} = \pm 23\ kN$$

M ger

$$P_i = \frac{-140}{7{,}58} \cdot r_i = -18{,}5 \cdot r_i$$

VERTIKALA

	a_{14}	b_{14}	a_{23}	b_{23}	c_{23}	d_{23}	c_{14}	d_{14}
Q	200	200	208	208	208	208	200	200
H	-23	-23	0	0	0	0	+23	+23
M	+7	-7	+22	+7	-7	-22	+7	-7
	184	170	230	215	201	186	230	216



$\tan\alpha = 0{,}286$
$\sin\alpha = 0{,}275$
$\cos\alpha = 0{,}96$
$\cos^2\alpha = 0{,}925$

$3\cdot 0 + 2\cdot 1 = 5e$

$e = 0{,}40\ m$

$y = 1{,}6\cdot 3{,}5 - 0{,}6 = 5m$

$\Sigma r_i^2 = (6\cdot 0{,}4^2 + 4\cdot 0{,}6^2)\,0{,}925 + 2\cdot 1^2 = 4{,}22\ m^2$

$M = 5500\cdot 0.2 - 160\cdot 5 = 300\ kNm$

Av Q

$$P_V = \frac{5500}{5 + 10\cdot 0{,}925} = 386\ kN$$

$$P_S = 386\cdot 0{,}96 = 371$$

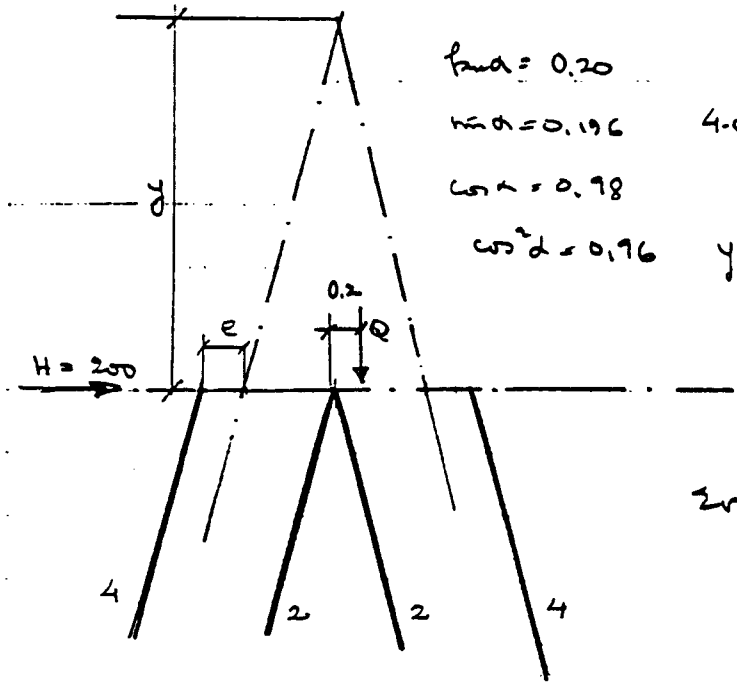
Av H

$$P_V = 0 \qquad P_S = \pm\frac{160}{10\cdot 0{,}275} = \pm 58\ kN$$

Av M

$$P_i = \frac{300}{4{,}22}\cdot r_i = 71\cdot r_i$$

Kraft av \ Pæle	b_2	c	d_2	a	b_{13}	d_{13}	e
	Vertikalt						
Q	386	386	386	371	371	371	371
H	0	0	0	− 58	− 58	+ 58	+ 58
M	− 71	0	+ 71	− 27	+ 41	− 41	+ 27
	315	386	457	286	354	388	456



$\tan\alpha = 0.20$

$\sin\alpha = 0.196$

$\cos\alpha = 0.98$

$\cos^2\alpha = 0.96$

$4 \cdot 0 + 2 \cdot 0{,}9 = 6 \cdot e$

$e = 0{,}3\,m$

$y = 0{,}6 \cdot 5 = 3\,m$

$M = 0{,}2 \cdot 2500 - 200 \cdot 3 = -100$

$\Sigma r^2 = \left(8 \cdot 0{,}3^2 + 4 \cdot 0{,}6^2\right) \cdot 0{,}96 = 2{,}07$

Av Q

$P_V = 0$

$$P_S = \frac{2500 \cdot \cos\alpha}{0 + 12 \cdot \cos^2\alpha} = \frac{2500 \cdot 0{,}98}{12 \cdot 0{,}96} = 213\ kN$$

Av H

$P_V = 0$

$$P_S = \pm \frac{1500}{12 \cdot 0{,}196} = -85\ kN$$

Av M

$$P_i = \frac{-100}{2{,}07} \cdot r_i = -48{,}3\, r_i$$

	a	b_{14}	b_{23}	c
Q	213	213	213	213
H	−85	−85	+85	+85
M	+14	−28	+28	−14
Σ	142	100	326	284

$M = H \cdot 0.6$ (motsurs)

$\tan\alpha = 0{,}20$
$\sin\alpha = 0{,}196$
$\cos\alpha = 0{,}98$
$\cos^2\alpha = 0{,}96$

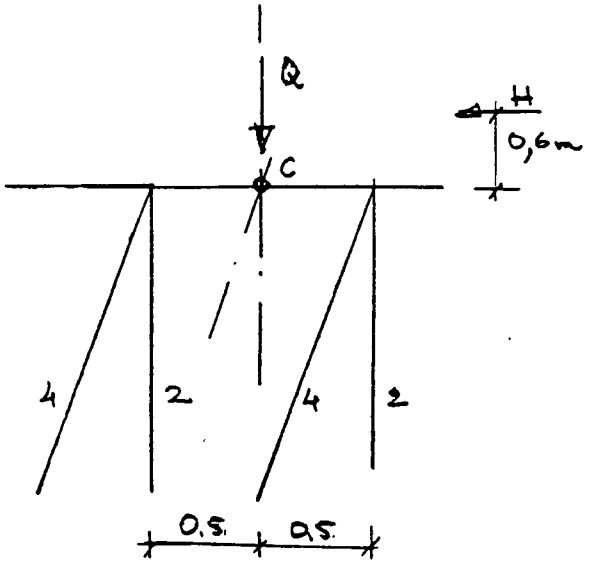
Ur Q erhålls

$$P_V = \frac{2160}{4} = 540\,kN \quad \Big| \; P_S = 0$$

Ur H erhålls

$$P_V = -\frac{H}{4 \cdot \tan\alpha} = -\frac{H}{4 \cdot 0.20}$$

$$P_S = \frac{H}{8 \cdot \sin\alpha} = \frac{H}{8 \cdot 0.196}$$



Ur M erhålls

$$P_i = \frac{-H \cdot 0.6}{2{,}92} \cdot r_i = -1{,}205\, r_i \cdot H$$

Mörka pålkraft i $b_{25} = 0$

$$\Sigma r_i^2 = 4 \cdot 0{,}5^2 + 8 \cdot 0{,}5^2 \cdot 0{,}96 = 2{,}92 \text{ m}^2$$

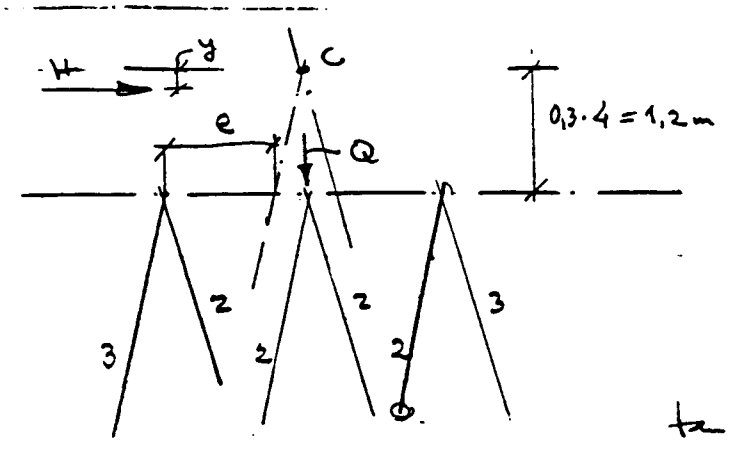
$$P = 540 - \frac{H}{0{,}8} - H \cdot \underbrace{0{,}205 \cdot 0{,}5}_{0.103} = 0$$

$$540 = 1{,}353\, H$$

$$\underline{H = 399 \text{ kN}}$$

VERTIKALA

Ur \ Pålc	a_{25}	b_{25}	a_{1346}	b_{1346}
Q	540	540	0	0
H	- 499	- 499	+ 254	+ 254
M	+ 40	- 40	+ 40	- 40
Σ	82	0	294	214



$3 \cdot 0 + 2 \cdot 2{,}1 + 2 \cdot 4{,}2 = 7 \cdot e$

$e = 1{,}8\ m$

$y = 1{,}2 - 1 = 0{,}2\ m$

$M = 825 \cdot 0{,}2 = 165\ kNm$

MOTURS !

$\tan\alpha = 0{,}25$

$\sin\alpha = 0{,}24$

$\cos\alpha = 0{,}97 \quad \cos^2\alpha = 0{,}94$

$$\Sigma r^2 = \left(6 \cdot 1{,}8^2 + 4 \cdot 0{,}3^2 + 4 \cdot 2{,}4^2\right) \cdot 0{,}94 = 40{,}3\ m^2$$

Av Q $\qquad P_s = \dfrac{Q}{14 \cdot 0{,}94} \cdot 0{,}97 = 0{,}0737 \cdot Q$

Av H $\qquad P_s = \pm \dfrac{825}{14 \cdot 0{,}24} = \pm 245{,}5\ kN$

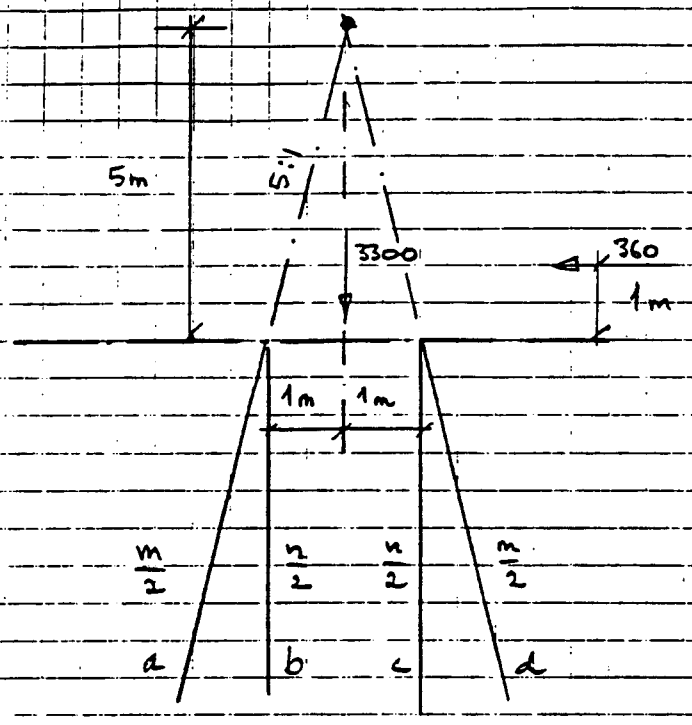
Av M $\qquad P_i = \dfrac{-165}{40{,}3} r_i = -4{,}09\ r_i$

$$0{,}0737\,Q - 245{,}5 - 4{,}09 \cdot 2{,}40 \cdot 0{,}97 = 0$$

$$Q = 3460\ kN$$

$$3460 = 6 \cdot 10 \cdot x \cdot 24$$

$$\underline{x = 2{,}40\ m}$$



$M = 360 \cdot 4 = 1440$ knäm (medurs)

$\Sigma r^2 = \frac{n}{2} \cdot 1^2 \cdot 2 = n \ (m^2)$

Av Q

$P_V = \frac{3300}{m+n}$ $\quad P_S = \frac{3300}{m+n}$

Av H

$P_V = 0$ $\quad P_S = \frac{360}{m \cdot 0{,}20} = \frac{1800}{m}$

Av M

$P_V = \frac{1440}{n} \cdot 1$ $\quad P_S = 0$

FÖR SNEDA PÅLAR GÄLLER

$$\begin{cases} P_a = \frac{3300}{m+n} + \frac{1800}{m} \leq 300 \\ P_d = \frac{3300}{m+n} - \frac{1800}{m} \geq 0 \end{cases}$$

GER $m = 12$ $\quad n = 10$

FÖR VERTIKALA PÅLAR GÄLLER

$$\begin{cases} P_b = \frac{3300}{m+n} - \frac{1440}{n} \geq 0 \\ P_c = \frac{3300}{m+n} + \frac{1440}{n} \leq 300 \end{cases}$$

GER $n = 9{,}6 \Rightarrow 10$ st $\quad m = 12$

9:1

EK 247 A

Perm. last, egenvekt $q_k = 4$ kN/m²

Variabel last, bunden $q_{bk} = 1$ kN/m² ($\psi = 1$)

fri $q_{fk} = 1.5$ kN/m² ($\psi = 0.5$)

Brottgränslast $q_d = 4 + 1{,}3(1 + 1.5) = \underline{7{,}25 \text{ kN/m}^2}$

Brukspr. last $q_d = 4 + 1 \cdot 1 + 0{,}5 \cdot 1.5 = \underline{5{,}75 \text{ kN/m}^2}$

B

Perm last, egenvekt $q_k = 4$ kN/m²

Variabel last, bunden $q_{bk} = 0$ kN/m² ($\psi = 1$)

fri $q_{fk} = 4$ kN/m² ($\psi = 0.5$)

Brottgränslast $q_d = 4 + 1.3 \cdot 4 = \underline{9.2 \text{ kN/m}^2}$

Brukgränslast. $q_d = 4 + 0{,}5 \cdot 4 = \underline{6 \text{ kN/m}^2}$

9.2

Ex 248

$q_k = 5$ kN/m²

a) $q_d = 5 + 1{,}3(1{,}0 + 1.5) = 8{,}25$ kN/m²

b) $q_d = 5 + 1{,}3(0 + 4) = 10{,}2$ kN/m²

c) $q_d = 5 + 1.3(0 + 2) = 7.6$ kN/m²

d) $q_d = 5 + 1.3(0 + 5) = 11.5$ kN/m²

e) $q_d = 5 + 1.3(0.5 + 1.5) = 7.6$ kN/m²

Ex 249

$s_0 = 3\ kN/m^2 \quad (\psi = 0{,}8) \qquad \mu = 0{,}8$

$s_k = 0{,}8 \cdot 3 = 2{,}4\ kN/m^2$

$g_k = 3{,}5\ kN/m^2$

Brottgränslast $\quad q_d = 3{,}5 + 1{,}3 \cdot 2{,}4 = \underline{6{,}62\ kN/m^2}$

Bruksgränslast $\quad q_d = 3{,}5 + 0{,}8 \cdot 2{,}4 = \underline{5{,}42\ kN/m^2}$

Ex 250

$s_0 = 2{,}5\ kN/m^2 \quad (\psi = 0{,}7)$

$\mu_1 = 0{,}4 \qquad \mu_2 = 0{,}55 \qquad s_{1k} = 0{,}4 \cdot 2{,}5 = 1{,}0\ kN/m^2$

$g_k = 0{,}5\ kN/m^2 \qquad s_{2k} = 0{,}55 \cdot 2{,}5 = 1{,}375\ kN/m^2$

Brottgränslast $\quad q_{1d} = 0{,}5 + 1{,}3 \cdot 1 = \underline{1{,}8\ kN/m^2}$

$q_{2d} = 0{,}5 + 1{,}3 \cdot 1{,}375 = \underline{2{,}29\ kN/m^2}$

Brukgränslast $\quad q_{1d} = 0{,}5 + 0{,}7 \cdot 1 = \underline{1{,}2\ kN/m^2}$

$q_{2d} = 0{,}5 + 0{,}7 \cdot 1{,}375 = \underline{1{,}46\ kN/m^2}$

Ex 251

$s_0 = 1\ kN/m^2 \quad (\psi = 0{,}6)$

$g_k = 0{,}59\ kN/m^2$

Brottgränslast $\quad 1{,}5 = 0{,}59 + 1{,}3 \cdot 1 \cdot \mu_2$

$\mu_2 = 0{,}70$

$$0{,}70 = 1{,}1 - \frac{1{,}1 \cdot (\beta - 30)}{30} \qquad \underline{\beta = 40{,}9^\circ}$$

Ex 252

$s_0 = 1 \qquad \mu = 0{,}8$

$q_k = 0{,}8\ kN/m^2 \qquad s_k = 0{,}8 \cdot 1 = 0{,}8\ kN/m^2$

ger $q_d = 0{,}8 + 1{,}3 \cdot 0{,}8 = \underline{1{,}04\ kN/m^2}$

Till bjälkarna

$$q_d = \frac{1{,}04 \cdot 10}{2} + 1{,}5 = \underline{10{,}7\ kN/m}$$

Till varje pelare

$$N_d = \frac{10{,}7 \cdot 12}{2} + 1 = \underline{65\ kN}$$

Ex 253

$s_0 = 2{,}5 \qquad \mu = 0{,}8$

$q_k = 0{,}6\ kN/m^2 \qquad s_k = 0{,}8 \cdot 2{,}5 = 2{,}0\ kN/m^2$

ger $q_d = 0{,}6 + 1{,}3 \cdot 2{,}0 = 3{,}2\ kN/m^2$

Till ovan

$q_d = 1{,}2 \cdot 3{,}2 = \underline{3{,}84\ kN/m}$

Till bjälkarna

$q_d = 3{,}1 \cdot 3{,}2 + 0{,}4 = \underline{10{,}3\ kN/m}$

Till pelarna

$$N_d = \frac{10{,}3 \cdot 9{,}6}{2} + 0{,}5 = \underline{50\ kN}$$

Ex 254

$q_{bk} = 0 \qquad q_{fk} = 5\ kN/m^2$

$q_k = 6\ kN/m^2$

ger $q_d = 6 + 1{,}3 \cdot 5 = \underline{12{,}5\ kN/m^2}$

Till bjälkarna

$q_d = 12{,}5 \cdot 5{,}6 + 8 = \underline{78\ kN/m}$

Till varje pelare

$\circlearrowleft B \quad R_A \cdot 7 - 78 \cdot 9 \cdot (4{,}5 - 2) = 0$

$\uparrow \quad R_A + R_B - 78 \cdot 9 = 0$

ger $R_A = 257\ kN \qquad R_B = 457\ kN$

+ egenvikt pelare $\underline{R_A = 261} \qquad \underline{R_B = 461\ kN}$

SNÖLAST

$s_0 = 3\ kN/m^2$ $(\psi = 0{,}8)$ $\mu = 0{,}8$

$q_k = 7\ kN/m^2$ $s_k = 0{,}8 \cdot 3 = 2{,}4\ kN/m^2$

$q_d = 7 + 1{,}3 \cdot 2{,}4 = 10{,}12\ kN/m^2$

Till pelare ① $N_{1d} = 10{,}12 \cdot 35 + 6 = \underline{360\ kN}$

NYTTIG LAST

$q_{bk} = 0{,}5$ $(\psi = 1)$ $q_{fk} = 1{,}5$ $(\psi = 0{,}33)$ $q_k = 7\ kN/m^2$

$q_d = 7 + 1{,}3 \cdot (0{,}5 + 1{,}5) = 7 + 2{,}6 = 9{,}6\ kN/m^2$

Lastgrupp 1, $\beta = 0{,}7$

Last till pelare ② $N_2 = (7 + 0{,}7 \cdot 2{,}6) \cdot 35 + 6 = \underline{315\ kN}$

SNÖLAST HUVUDLAST

$N_1 = 360\ kN$

$q_v = 7 + 1 \cdot 1{,}5 + 0{,}33 \cdot 1{,}5 = 7 + 1{,}0 = 8{,}0\ kN/m^2$

$N_{2v} = (7 + 0{,}7 \cdot 1{,}0) \cdot 35 + 6 = 276\ kN$

$N_1 + N_{2v} = 360 + 276 = \underline{636\ kN = N_{2d}}$ AVGÖRANDE!

NYTTIG LAST HUVUDLAST

$N_2 = 315\ kN$

Snölasten reduceras

$q_v = 7 + 0{,}8 \cdot 2{,}4 = 8{,}92\ kN/m^2$

$N_{1v} = 8{,}92 \cdot 35 + 6 = 318\ kN$

$N_{1v} + N_2 = 318 + 315 = \underline{633\ kN}$

SNÖLAST

$s_0 = 2\ kN/m^2 (\psi = 0{,}7)$ $\mu = 0{,}8$

$g_k = 4{,}5\ kN/m^2$ $s_k = 0{,}8 \cdot 2 = 1{,}6\ kN/m^2$

$q_d = 4{,}5 + 1{,}3 \cdot 1{,}6 = 6{,}58\ kN/m^2$

Till pelare (1) $N_{1d} = 6{,}58 \cdot 48 + 5 = \underline{321\ kN}$

NYTTIG LAST

$q_{bk} = 1{,}0$ $(\psi = 1)$ $q_{fk} = 1{,}5$ $(\psi = 0{,}5)$ $g_k = 6\ kN/m^2$

$q_d = 6 + 1{,}3(1 + 1{,}5) = 6 + 3{,}25 = 9{,}25\ kN/m^2$

Lastgrupp 2, $\beta = 0{,}7$

Last till pelare (2) eller (3)

$N_2 = N_3 = (6 + 0{,}7 \cdot 3{,}25) \cdot 48 + 5 = \underline{402\ kN}$

SNÖLASTEN HUVUDLAST

$N_1 = \underline{321\ kN}$

$q_V = 6 + 1 \cdot 1 + 0{,}5 \cdot 1{,}5 = 6 + 1{,}75 = 7{,}75\ kN/m^2$

$N_{2V} = N_{3V} = (6 + 0{,}7 \cdot 1{,}75) \cdot 48 + 5 = \underline{352\ kN}$

$N_1 + N_{2V} + N_{3V} = 321 + 2 \cdot 352 = \underline{1025\ kN}$

EN NYTTIG LAST HUVUDLAST

$N_2 = \underline{402\ kN}$ $N_{3V} = \underline{352\ kN}$

Snölasten reduceras

$q_V = 4{,}5 + 0{,}7 \cdot 1{,}6 = 5{,}62\ kN/m^2$

$N_{1V} = 5{,}62 \cdot 48 + 5 = \underline{275\ kN}$

$N_{1V} + N_2 + N_{3V} = 275 + 402 + 352 = \underline{1029\ kN = N_{3d}}$

AVGÖRANDE

a) SNÖLAST $s_0 = 4\ kN/m^2\ (\varphi = 0{,}8)$ $\mu = 0{,}8$ **Ex 257**

$\gamma_k = 1$ $s_k = 0{,}8 \cdot 4 = 3{,}2\ kN/m^2$

$q_d = 1 + 1{,}3 \cdot 3{,}2 = \underline{5{,}16\ kN/m^2}$

Till bjälkarna: $q_d = 5{,}16 \cdot 5 + 1{,}5 = \underline{27{,}3\ kN/m}$

Till pelarna i plan 1 tr. $N_{1d} = \frac{27{,}3 \cdot 8}{2} + 2 = \underline{111\ kN}$

b) NYTTIG LAST $q_{bk} = 1\ (\psi = 1)$ $q_{fk} = 1{,}5\ (\psi = 0{,}5)$

$q_k = 2$

$q_d = 2 + 1{,}3\,(1 + 1{,}5) = 2 + 3{,}25 = \underline{5{,}25\ kN/m^2}$

Lastgrupp 2, $\beta = 0{,}7$

$q_{d\beta} = 2 + 0{,}7 \cdot 3{,}25 = 4{,}28\ kN/m^2$

Till bjälkarna $q_d = 4{,}28 \cdot 5 + 4{,}5 = \underline{25{,}9\ kN/m}$

Till pelarna i plan 1 $N_{2d} = \frac{25{,}9 \cdot 8}{4} + 2 = \underline{106}$

c) SNÖLASTEN HUVUDLAST

$N_1 = \underline{111\ kN}$

$q_v = 2 + 1 \cdot 1 + 0{,}5 \cdot 1{,}5 = 2 + 1{,}75 = 3{,}75\ kN/m^2$

$q_{v\beta} = 2 + 0{,}7 \cdot 1{,}75 = 3{,}23\ kN/m^2$

Till bjälkarna $q_v = 3{,}23 \cdot 5 + 4{,}5 = 20{,}6\ kN/m$

Till pelarna $N_{2v} = \frac{20{,}6 \cdot 8}{2} + 2 = \underline{84\ kN}$

$N_1 + N_{2v} = 111 + 84 = \underline{195\ kN = N_{2d}}$ AVGÖRANDE

NYTTIG LAST HUVUDLAST

$\underline{N_2 = 106\ kN}$

$q_v = 1 + 0{,}8 \cdot 3{,}2 = 3{,}56\ kN/m^2$

Till bjälkarna $q_v = 3{,}56 \cdot 5 + 1{,}5 = 19{,}3\ kN/m$

Till pelarna $N_{1v} = \frac{19{,}3 \cdot 8}{2} + 2 = 79\ kN$

$N_{1v} + N_2 = 79 + 106 = \underline{185\ kN}$

SNÖLAST

$s_0 = 1.5\ kN/m^2 \quad (\psi = 0{,}7)$ **Ex 258**

$\mu_1 = 0{,}8 \qquad \mu_2 = 1{,}04$ ger

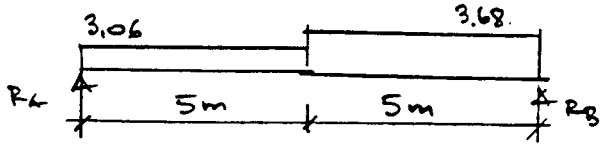
$s_{1k} = 0{,}8 \cdot 1.5 = 1{,}2\ kN/m^2 \qquad s_{2k} = 1{,}04 \cdot 1{,}5 = 1{,}56\ kN/m^2$

$g_k = 1 + \frac{0{,}6}{1{,}2} = 1{,}5\ kN/m^2$

Betrakta 1 m bredd

$q_{1d} = 1{,}5 + 1{,}3 \cdot 1{,}2 = 3{,}06\ kN/m^2$

$q_{2d} = 1{,}5 + 1{,}3 \cdot 1{,}56 = 3{,}53\ kN/m^2$



$\circlearrowleft A \quad 3{,}06 \cdot 5 \cdot 2{,}5 + 3{,}53 \cdot 5 \cdot 7{,}5 - R_{B1} \cdot 10 = 0 \qquad \underline{R_{B1} = 17{,}1\ kN/m}$

Om snölasten inte är huvudlast erhålles vanlig last

$q_{1v} = 1{,}5 + 0{,}7 \cdot 1{,}2 = 2{,}34\ kN/m^2$

$q_{2v} = 1{,}5 + 0{,}7 \cdot 1{,}56 = 2{,}59\ kN/m^2$

$\circlearrowleft A \quad 2{,}34 \cdot 5 \cdot 2{,}5 + 2{,}59 \cdot 5 \cdot 7{,}5 - R_{B1v} = 0 \qquad \underline{R_{B1v} = 12{,}6\ kN/m}$

NYTTIG LAST

$q_{bk} = 0 \qquad q_{fk} = 4\ kN/m^2 \quad (\psi = 0{,}5) \qquad g_k = 1{,}5\ kN/m^2$

$q_d = 1{,}5 + 1{,}3 \cdot 4 = 6{,}7\ kN/m^2$

Till väggen $R_{B2} = 6{,}7 \cdot 2{,}5 = \underline{16{,}8\ kN/m}$

Om nyttig last inte är huvudlast

$q_v = 1{,}5 + 0{,}5 \cdot 4 = 3{,}5\ kN/m^2$

Till väggen $R_{B2v} = 3{,}5 \cdot 2{,}5 = \underline{8{,}8\ kN/m}$

VÄGGAR

$R_{Bg} = 2{,}8 \cdot 1 + 2{,}8 \cdot 2 = \underline{8{,}4\ kN}$

Fall 1 (snölasten huvudlast) $R_B = R_{B1} + R_{B2v} + R_g =$

$= 17{,}1 + 8{,}8 + 8{,}4 = 34{,}3\ kN/m$

Fall 2 (nyttig last huvudlast) $R_B = R_{B1v} + R_{B2} + R_g =$

$= 12{,}6 + 16{,}8 + 8{,}4 = \underline{37{,}8\ kN/m}$

AVGÖRANDE

Utbredd last dimensionerande

$q_d = 7 + 17 + 1{,}3 \cdot 26 = 57{,}8$ kN/m

$F_v = 110 + 0{,}5 \cdot 140 = 180$ kN

$R_B \cdot 8 - 57{,}8 \cdot 10 \cdot 5 - 180 \cdot 10 = 0$ $\qquad R_B = 586$ kN

Punktlasten dimensionerande

$q_v = 7 + 17 + 0{,}5 \cdot 26 = 37$ kN/m

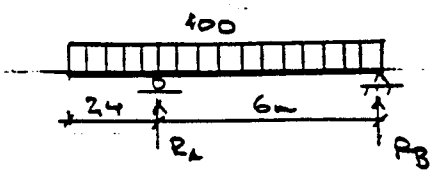
$F_d = 110 + 1{,}3 \cdot 140 = 292$ kN

$R_B \cdot 8 - 37 \cdot 10 \cdot 5 - 292 \cdot 10 = 0$ $\qquad R_B = 596$ kN

ANGÖRANDE

$N_d = 596 + 10 = 606$ kN

I



$R_A = \frac{100 \cdot 8,4 \cdot 4,2}{6} = 588$ kN

$R_{AV} = 240$ kN $\quad R_{AH} = 348$ kN

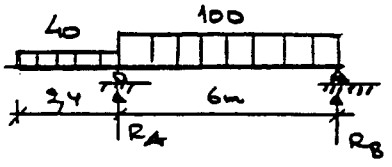
$R_B = 840 - 588 = 252$ kN

$M_B = -100 \cdot 2,4 \cdot 1,2 = -288$ kNm

$M^o_{CA} = \frac{100 \cdot 2,4^2}{8} = 72$ kNm $\quad M^o_{AB} = \frac{100 \cdot 6^2}{8} = 450$ kNm

---

II



$R_A = \frac{100 \cdot 6 \cdot 3 + 40 \cdot 2,4 \cdot 7,2}{6} = 415,2$

$R_{AV} = 96 \quad R_{AH} = 319,2$

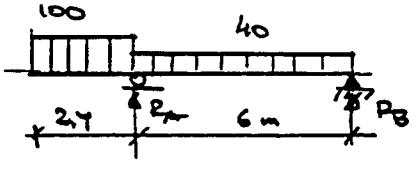
$R_B = 40 \cdot 2,4 + 600 - 415,2 = 280,8$ kN

$M_B = -40 \cdot 2,4 \cdot 1,2 = -115$ kNm

$M^{max}_{AB} = \frac{280,8^2}{200} = 394 \quad M^o_{AB} = 450$ kNm

---

III



$R_A = \frac{40 \cdot 6 \cdot 3 + 100 \cdot 2,4 \cdot 7,2}{6} = 408$

$R_{AV} = 240 \quad R_{AH} = 168$ kN

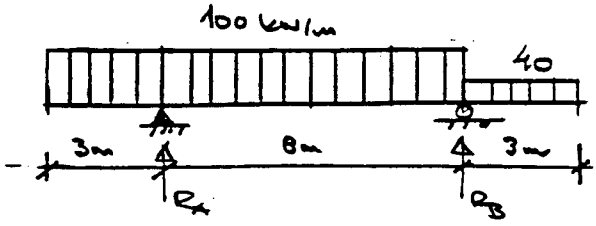
$R_B = 240 + 240 - 408 = 72$ kN

$M_B = -288$ kNm

$M^o_{CA} = 72$ kNm $\quad M^o_{AB} = \frac{40 \cdot 6^2}{8} = 180$ kNm

---

I

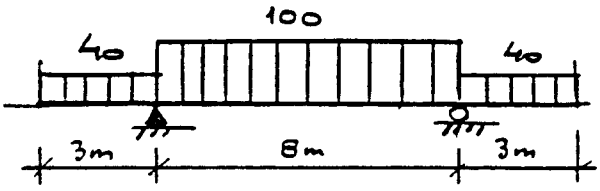


$$R_A = \frac{100 \cdot 11 \cdot 5{,}5 - 40 \cdot 3 \cdot 1{,}5}{8} = 734$$

$R_{AV} = 300$ $\quad R_{AH} = 434$ kN

$R_B = 1100 + 170 - 734 = 486$ kN $\quad R_{BV} = 366$ kN $\quad R_{BH} = 120$ kN

II

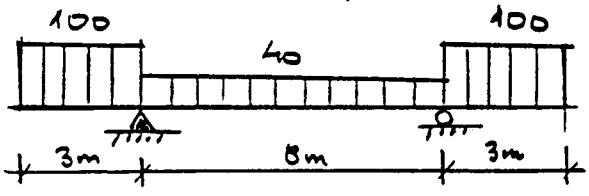


$$M_A = M_B = -\frac{40 \cdot 3^2}{2} = -180$$

$$M^0_{AB} = \frac{100 \cdot 8^2}{8} = 800$$

$$M^{max}_{AB} = 800 - 180 = 620 \text{ kNm}$$

III

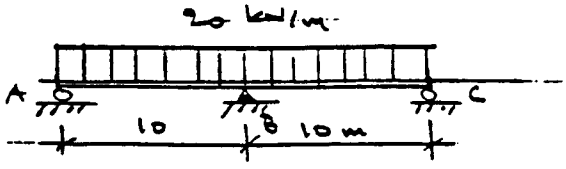


$$M_A = M_B = -\frac{100 \cdot 3^2}{2} = -450 \text{ kNm}$$

$$M^0_{AB} = \frac{40 \cdot 8^2}{8} = 320 \text{ kNm}$$

$$M_{AB} = 320 - 450 = -130 \text{ kNm}$$

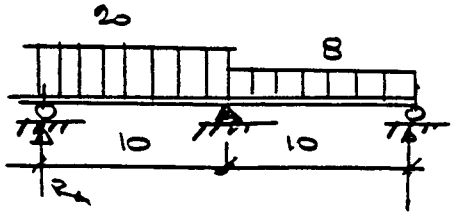
I



$M_B = -0{,}125 \cdot 20 \cdot 10^2 = -250$ kNm

$M^o_{AB} = M^o_{BC} = 250$ kNm

II



$M_B = -0{,}125 \cdot 8 \cdot 10^2 - 0{,}0625 \cdot 12 \cdot 10^2$

$= -175$ kNm

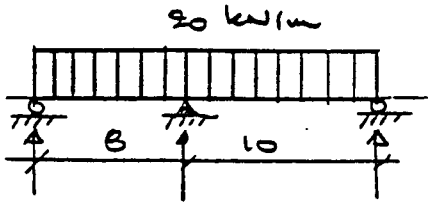
$M^o_{AB} = 250 \qquad M^o_{BC} = \frac{8 \cdot 10^2}{8} = 100$ kN

$R_A = \frac{20 \cdot 10}{2} - \frac{175}{10} = 82{,}5$ kN

$M^{max}_{AB} = \frac{82{,}5^2}{2 \cdot 20} = 170$ kNm

Ex 263

I



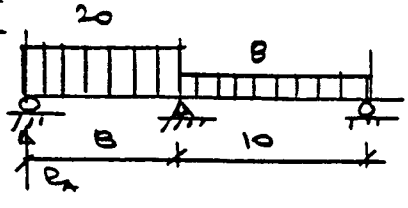
$\frac{8}{6EI}\left(\frac{20 \cdot 8^2}{4} + 2M_B\right) = \frac{10}{6EI}\left(-\frac{20 \cdot 10^2}{4} - 2M_B\right)$

$36 M_B = -(2560 + 5000) \qquad M_B = -210$ kNm

$M^o_{AB} = \frac{20 \cdot 8^2}{8} = 160 \qquad M^o_{BC} = \frac{20 \cdot 10^2}{8} = 250$ kNm

---

II



$\frac{8}{6EI}\left(\frac{20 \cdot 8^2}{4} + 2M_B\right) = \frac{10}{6EI}\left(-\frac{8 \cdot 10^2}{4} - 2M_B\right)$

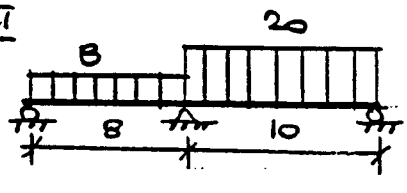
$36 M_B = -(2560 + 2000) \qquad M_B = -126{,}7$

$M^o_{AB} = 160 \qquad M^o_{BC} = \frac{8 \cdot 10^2}{8} = 100$ kNm

$R_A = \frac{20 \cdot 8}{2} - \frac{126{,}7}{8} = 64{,}2$ kN $\qquad M^{max}_{AB} = \frac{64{,}2^2}{2 \cdot 20} = 103$ kNm

---

III



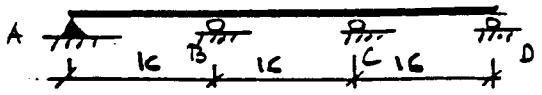
$\frac{8}{6EI}\left(\frac{8 \cdot 8^2}{4} + 2M_B\right) = \frac{10}{6EI}\left(-\frac{20 \cdot 10^2}{4} - 2M_B\right)$

$36 M_B = -(1024 + 5000) \qquad M_B = -167{,}3$

$M^o_{AB} = \frac{8 \cdot 8^2}{8} = 64 \qquad M^o_{BC} = 250$ kNm

$R_C = \frac{20 \cdot 10}{2} - \frac{167{,}3}{10} = 83{,}3$ kN $\qquad M^{max}_{BC} = \frac{83{,}3^2}{2 \cdot 20} = 173$ kNm

Ex 264

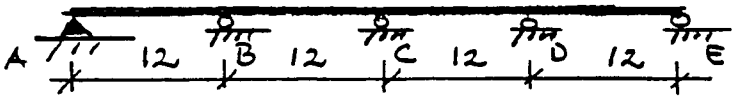


$$M_B^{max} = M_C^{max} = -0{,}100 \cdot 10 \cdot 16^2 - 0{,}117 \cdot 15 \cdot 16^2 = \underline{-705 \text{ kNm}}$$

$$M_{AB}^{max} = M_{CD}^{max} = 0{,}080 \cdot 10 \cdot 16^2 + 0{,}101 \cdot 15 \cdot 16^2 = \underline{593 \text{ kNm}}$$

$$M_{BC}^{max} = 0{,}025 \cdot 10 \cdot 16^2 + 0{,}075 \cdot 15 \cdot 16^2 = \underline{352 \text{ kNm}}$$

Ex 265



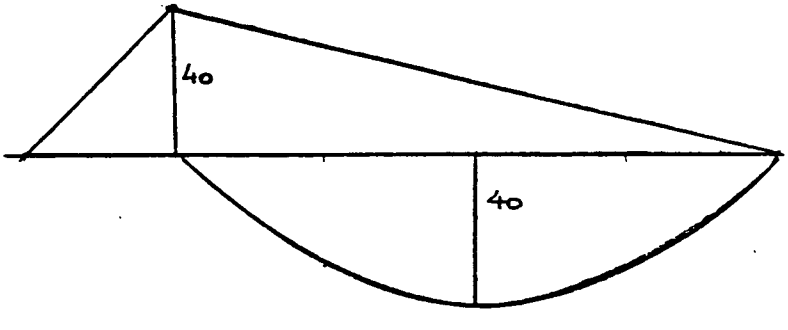
$$M_B^{max} = M_D^{max} = -0{,}107 \cdot 10 \cdot 16^2 - 0{,}121 \cdot 15 \cdot 16^2 = \underline{-415 \text{ kNm}}$$

$$M_C^{max} = -0{,}077 \cdot 10 \cdot 16^2 - 0{,}107 \cdot 15 \cdot 16^2 = \underline{-333 \text{ kNm}}$$

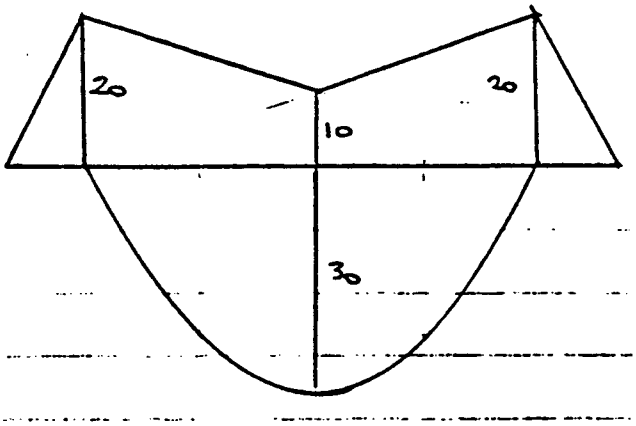
$$M_{AB}^{max} = 0{,}077 \cdot 10 \cdot 12^2 + 0{,}100 \cdot 15 \cdot 16^2 = \underline{327 \text{ kNm}}$$

$$M_{BC}^{max} = 0{,}036 \cdot 10 \cdot 12^2 + 0{,}081 \cdot 15 \cdot 16^2 = \underline{225 \text{ kNm}}$$

Ex 266

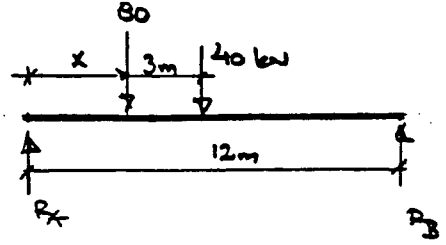


Ex 267



halvera höjdskalan

Ex 268



$$\circlearrowleft B \quad R_A \cdot 12 - 80(12-x) - 40(9-x) = 0$$

$$R_A = 110 - 10x$$

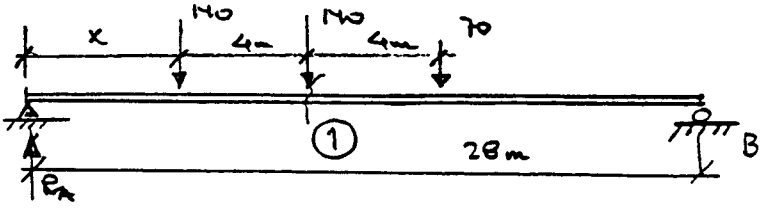
$$M_x = R_A \cdot x = 110x - 10x^2$$

$$\frac{dM}{dx} = 110 - 20x \qquad \text{ger } \underline{x = 5{,}5\,m}$$

$$M_{max} = 110 \cdot 5{,}5 - 10 \cdot 5{,}5^2 = 302{,}5 \text{ kNm} \sim \underline{303 \text{ kNm}}$$

det betyder är 5,5 m från stöd A

Ex 269



$$\circlearrowleft B \quad R_A \cdot 28 - 140(28-x) - 140(24-x) - 70(20-x) = 0$$

$$28 \cdot R_A = 8680 - 350x \qquad R_A = 310 - 12{,}5x$$

Största momentet under mitt lasten

$$M_1 = R_A \cdot (x+4) - 140 \cdot 4 =$$

$$= (310 - 12{,}5x)(x+4) - 560$$

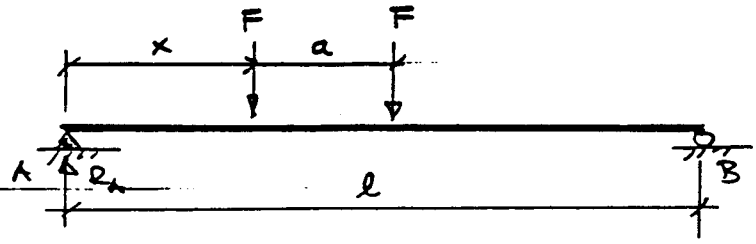
$$= 310x - 12{,}5x^2 + 1240 - 50x - 560$$

$$= 680 - 12{,}5x^2 + 260x$$

$$\frac{dM_1}{dx} = -25x + 260 = 0 \qquad \underline{x = 10{,}4\,m}$$

$$M_1^{max} = 680 - 12{,}5 \cdot 10{,}4^2 + 260 \cdot 10{,}4 = \underline{2032 \text{ kNm}}$$

Ex 270



$$\circlearrowright B \quad R_A \cdot l - F(l-x) - F(l-x-a) = 0$$

$$R_A l = Fl - Fx + Fl - Fx - Fa$$

$$R_A = \frac{F}{l}(2l - 2x - a)$$

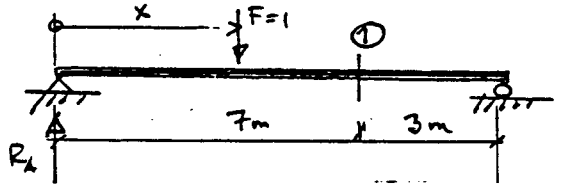
$$M_{max} = R_A \cdot x = \frac{F}{l}(2lx - 2x^2 - ax)$$

$$\frac{dM}{dx} = \frac{F}{l}(2l - 4x - a) = 0$$

vavar $\quad x = \frac{2l-a}{4} = \underline{\frac{l}{2} - \frac{a}{4}}$

d.v.s. ena hjulet skall befinna sig $\frac{a}{4}$ från balkmitt.

Ex 271



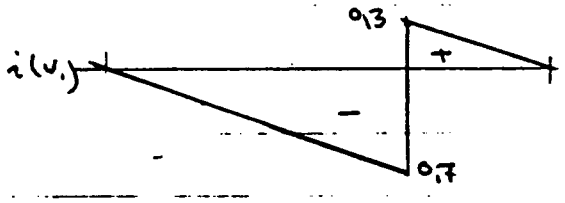
$$\circlearrowright B \quad R_A \cdot 10 - F(10-x) = 0$$

$$R_A = F\left(1 - \frac{x}{10}\right)$$

$\underline{0 \le x \le 7 \text{ m}}$

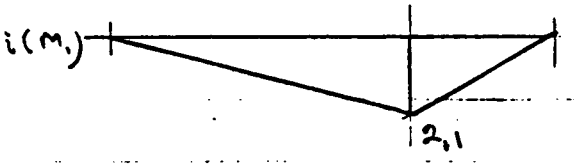
$$\downarrow \quad -F\left(1-\frac{x}{10}\right) + F + V_1 = 0$$

$$V_1 = -\frac{Fx}{10} \qquad \underline{i(V_1) = -\frac{x}{10}}$$



$$\circlearrowleft \quad M_1 - F\left(1-\frac{x}{10}\right)\cdot 7 + F(7-x) = 0$$

$$M_1 = 0{,}3Fx \qquad \underline{i(M_1) = 0{,}3x}$$



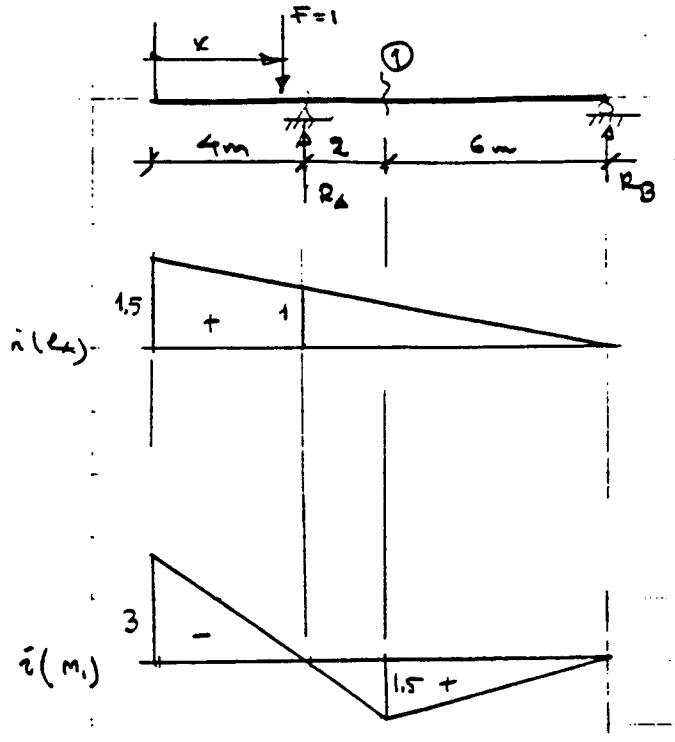
$\underline{7 \le x \le 10 \text{ m}}$

$$\downarrow \quad -F\left(1-\frac{x}{10}\right) + V_1 = 0$$

$$V_1 = F\left(1-\frac{x}{10}\right) \qquad \underline{i(V_1) = 1 - \frac{x}{10}}$$

$$\circlearrowleft \quad M_1 - F\left(1-\frac{x}{10}\right)\cdot 7 = 0$$

$$M_1 = 7F - 0{,}7Fx \qquad \underline{i(M_1) = 7 - 0{,}7x}$$



$$\circlearrowleft B \quad R_A \cdot 8 - F(12-x) = 0$$

$$R_A = \frac{F}{8}(12-x)$$

$$i(R_A) = \frac{12-x}{8}$$

$x=0$ per $i = 1.5$

$x = 12$ per $i = 0$

$0 \le x \le 6\,m$

$$M_1 = \frac{F(12-x)2}{8} - F(6-x) =$$

$$= 3F - \frac{xF}{4} - 6F + Fx$$

$$i(M_1) = \frac{3x}{4} - 3$$

$$\begin{cases} x=0 \text{ per } i = -3 \\ x = 6 \text{ per } i = +1.5 \end{cases}$$

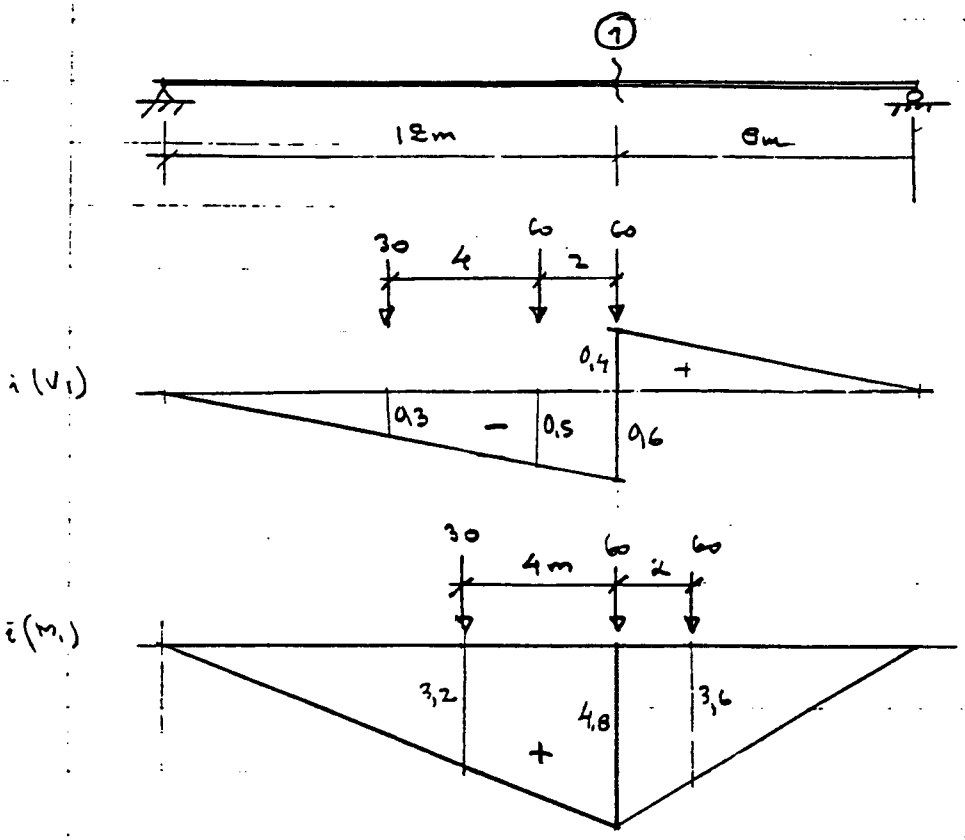
$6 \le x \le 12\,m$

$$M_x = \frac{F\cdot(12-x)\cdot 2}{8} = \frac{F(12-x)}{4}$$

$$i(M_1) = \frac{12-x}{4} = 3 - \frac{x}{4}$$

$$\begin{cases} x=6 \text{ per } i = 1,5 \\ x=12 \text{ per } i = 0 \end{cases}$$

Ex 273.



$x = 0$ o $x = 20$ ger

$V_1 = M_1 = 0$

$x = 12$ ger

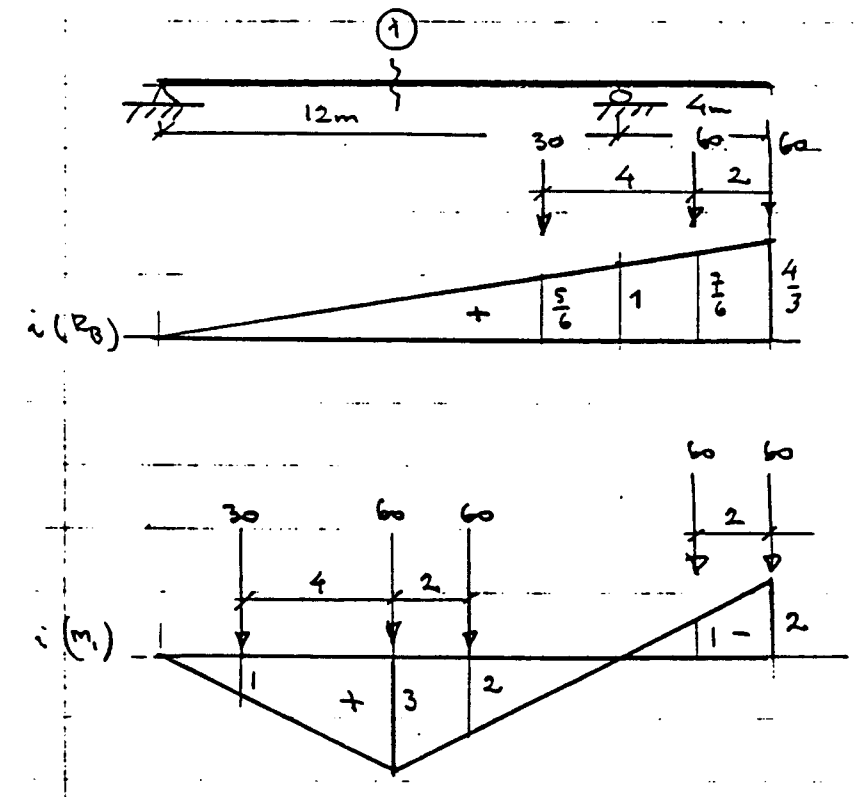
$V_1 = 0.4$ och 0.6

$M_1 = 0.4 \cdot 12 = 4{,}8$ m

$V_{max} = 60 \cdot 0{,}6 + 60 \cdot 0{,}5 + 30 \cdot 0{,}3 = \underline{75 \text{ kN}}$

$M_{max} = 60 \cdot 4.8 + 60 \cdot 3{,}6 + 30 \cdot 3{,}2 = \underline{600 \text{ kNm}}$

Ex 274



$x = 0$ ger $R_B = 0$

$x = 12$ ger $R_B = \frac{4}{3}$

$R_B^{max} = 60 \cdot \frac{4}{3} + 60 \cdot \frac{7}{6} + 30 \cdot \frac{5}{6} = \underline{175 \text{ kN}}$

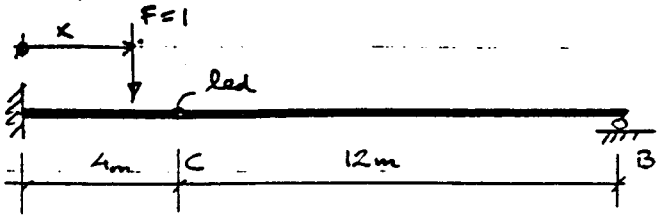
$x = 0$, $x = 12$ ger $M_1 = 0$

$x = 6$ $R_A = 0.5$ $M_1 = 3$ m

$M_1^{+max} = 60 \cdot 3 + 60 \cdot 2 + 30 \cdot 1 = \underline{330 \text{ kNm}}$

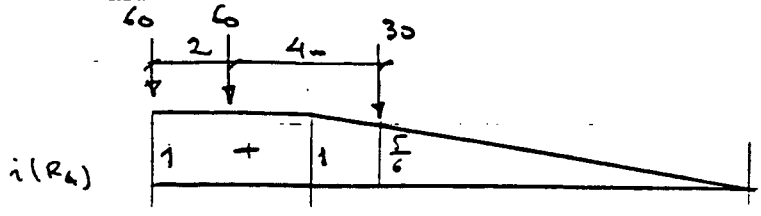
$M_1^{-max} = -60 \cdot 2 - 60 \cdot 1 = \underline{-180 \text{ kN}}$

Ex 275



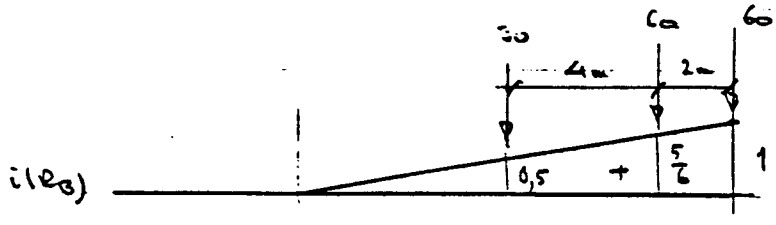
$x=0 \quad R_A = 1 \quad R_B = 0$

$M_A = 0$



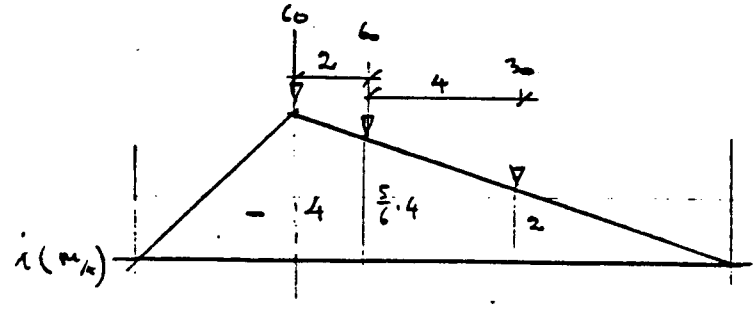
$x=4 \quad R_A = 1 \quad R_B = 0$

$M_A = 4$



$x=16 \quad R_A = 0 \quad R_B = 1$

$M_A = 0$



$R_A^{max} = 60\cdot 1 + 60\cdot 1 + 30\cdot\frac{5}{6}$

$= \underline{145\ kN}$

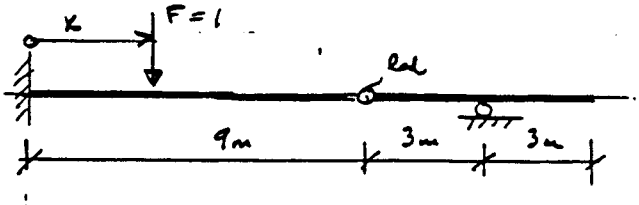
$R_B^{max} = 60\cdot 1 + 60\cdot\frac{5}{6} + 30\cdot\frac{1}{2}$

$= \underline{125\ kN}$

$M_A^{max} = -60\cdot 4 - 60\cdot\frac{5\cdot 4}{6} - 30\cdot 2$

$= \underline{-500\ kN}$

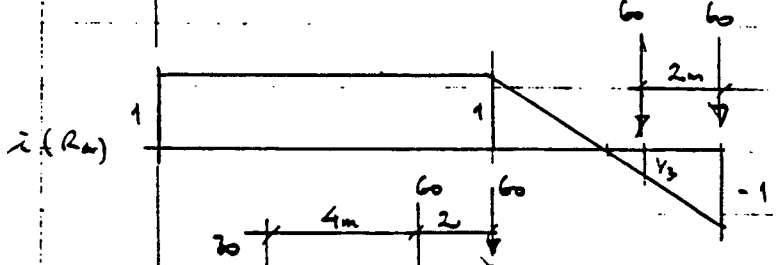
Ex 276



$x=0 \quad R_A = 1 \quad M_A = 0$

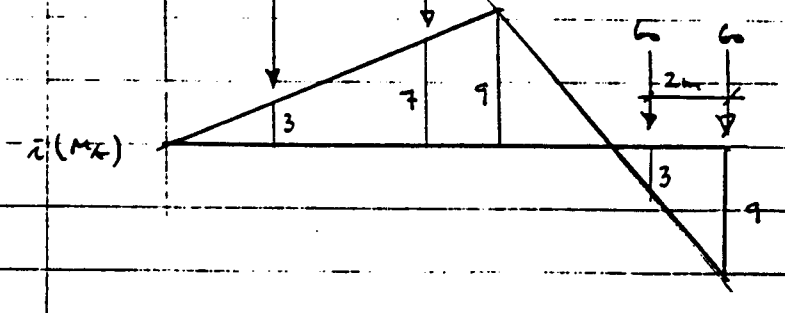
$x=9 \quad R_A = 1 \quad M_A = 9$

$x=15 \quad R_A = -1 \quad M_A = -9$



$R_A^{+max} = 60\cdot 1 + 60\cdot 1 + 30\cdot 1 = \underline{150\ kN}$

$R_A^{-max} = -60\cdot 1 - 60\cdot\frac{1}{3} = \underline{-80\ kN}$

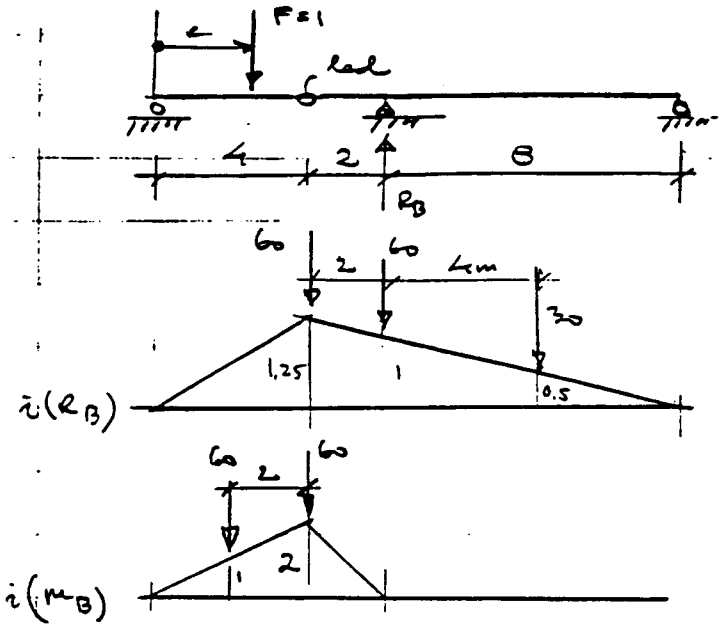


$M_A^{-max} = -60\cdot 9 - 60\cdot 7 - 30\cdot 3 =$

$= \underline{-1050\ kNm}$

$M^{+max} = 60\cdot 9 + 60\cdot 3 = \underline{720\ kNm}$

Ex 277



$x=0 \quad R_B=0 \quad M_B=0$

$x=4 \quad R_B=1.25 \quad M_B=2$

$x=6 \quad R_B=1 \quad M_B=0$

$x=14 \quad R_B=0 \quad M_B=0$

$R_B^{max} = 60\cdot 1{,}25 + 60\cdot 1 + 30\cdot 0{,}5 =$

$= \underline{150 \text{ kN}}$

$M_B^{max} = -60\cdot 2 - 60\cdot 1 = \underline{-180 \text{ kN}}$

Ex 278

Varje längdenhet $\Rightarrow \frac{5}{20} = 0{,}25$ m 2m $\Rightarrow$ 8 enheter

$x = 0{,}20\,l = 1.0$ m och $0{,}60\,l = 3{,}0$ m ger

$M_B^{max} = (-0{,}13 + 0.095)\cdot 40\cdot 5 = \underline{-45 \text{ kNm}}$

$x = 0{,}5\,l = 2{,}5$ m och $0{,}9\,l = 4{,}5$ m ger

$M_1^{max} = (0{,}125 + 0{,}005)\cdot 40\cdot 5 = \underline{+26 \text{ kNm}}$

Ex 279

Varje längdenhet $\Rightarrow$ 1 m 6 m $\Rightarrow$ 6 enheter

$x = 0{,}70\,l = 7$ m och $1{,}30\,l = 13$ m ger

$M_B^{-max} = -(0{,}095 + 0{,}077)\cdot 60\cdot 10 = \underline{-103 \text{ kNm}}$

$x = 2{,}2\,l = 22$ m och $2{,}8\,l = 28$ m ger

$M_B^{+max} = +(0{,}019 + 0{,}013)\cdot 60\cdot 10 = \underline{+19 \text{ kNm}}$

$x = 0{,}40\,l = 4$ m ger $M_1^{+max} = 0{,}204\cdot 60\cdot 10 = \underline{122 \text{ kNm}}$

$x = 1{,}2\,l = 12$ m och $x = 1{,}8\,l = 18$ m ger

$M_1^{-max} = -(0{,}026 + 0{,}013)\cdot 60\cdot 10 = \underline{-23 \text{ kNm}}$

Ex 280

Varje längdenhet = 1,5 m 6 m = 4 enheter

$x = 0{,}3l = 4{,}5\,m$ och $0{,}75l = 10{,}5\,m$ ger

$M_B^{max} = -(0{,}068 + 0{,}090) \cdot 60 \cdot 15 = -142\ kNm$

$x = 0{,}4l = 6\,m$ och $x = 0{,}8l = 12\,m$ ger

$M_1^{+max} = +(0{,}206 + 0{,}052)\ 60 \cdot 15 = 232\ kNm$

$x = 1{,}3l = 19{,}5\,m$ och $x = 1{,}7l = 25{,}5\,m$ ger

$M_1^{-max} = -(0{,}036 + 0{,}027) \cdot 60 \cdot 15 = -57\ kNm$

Ex 281

Varje längdenhet = 1,2 m 6 m = 5 enheter

$x = 0{,}7l = 8{,}4\,m$ och $1{,}2l = 14{,}4\,m$ ger

$M_B^{-max} = -(0{,}095 + 0{,}064) \cdot 60 \cdot 12 = -115\ kNm$

$x = 0{,}4l = 4{,}8\,m$ och $0{,}9l = 10{,}8\,m$ ger

$M_1^{+max} = +(0{,}204 + 0{,}022) \cdot 60 \cdot 12 = +163\ kNm$

$x = 1{,}2l = 14{,}4\,m$ och $1{,}7l = 20{,}4\,m$ ger

$M_1^{-max} = -(0{,}025 + 0{,}020) \cdot 60 \cdot 12 = -32\ kNm$

$x = 2{,}2l = 26{,}4\,m$ och $x = 2{,}7l = 32{,}4\,m$

$M_B^{+max} = +(0{,}019 + 0{,}018) \cdot 60 \cdot 12 = 27\ kNm$